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Preliminary Study of Electron Emission for Use in the PIC Portion of MAFIA

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June 2001

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Preliminary Study of Electron Emission for use in the PIC Portion of MAFIA

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Abstract

This memorandum summarizes a study undertaken to apply the program MAFIA to the modeling of an electron gun in a traveling wave tube (TWT). The basic problem is to emit particles from the cathode in the proper manner. The electrons are emitted with the classical Maxwell-Boltzmann (M-B) energy distribution; and for a small patch of emitting surface; the distribution with angle obeys Lambert's law. This states that the current density drops off as the cosine of the angle from the normal. The motivation for the work is to extend the analysis beyond that which has been done using older codes. Some existing programs use the Child-Langmuir, or 3/2 power law, for the description of the gun. This means the current varies as the 3/2 power of the anode voltage. The proportionality constant is termed the perveance of the gun. This is limited, however, since the 3/2 variation is only an approximation. Also, if the cathode is near saturation, the 3/2 law definitely will not hold. In most of the older codes, the electron beam is decomposed into current tubes, which imply laminar flow in the beam; even though experiments show the flow to be turbulent. Also, the proper inclusion of noise in the beam is not possible. These older methods of calculation do, however, give reasonable values for parameters of the electron beam and the overall gun,

and these values will be used as the starting point for a more precise particle-in-cell (PIC) calculation.

To minimize the time needed for a given computer run, all beams will use the same number of particles in a simulation. This is accomplished by varying the mass and charge of the emitted particles (macroparticles) in a certain manner, to be consistent with the desired beam current.

Chapter 1, Emission Modeling, gives a general overview of how an emission calculation should proceed, and develops equations that adhere to the basic constraints of the emission process. It ends by stating the number of particles for a simulation using this technique would be prohibitive. The remaining sections show alternate paths that will be pursued, to obtain outputs that will be similar to published experimental reports. Chapter 2, Low Voltage Diode, gives the analytical result for a 1-dimensional planar diode with only 1.172 volts applied to the anode. This example demonstrates the magnitudes of the numbers involved. Then Chapter 3, Particle Enumeration, shows how to modify the particle mass and charge to reduce the run time to a reasonable value. Finally, the probability density for a reasonable model of an electron beam in other parts of the TWT is developed for analysis purposes. Appendix I gives a proof of constraints used in earlier sections.

Introduction

This memorandum summarizes the study that was undertaken to determine if the electromagnetic solver, MAFIA, could be used to model the electron gun of a traveling wave tube. The preliminary conclusion is that it can be used; and the outline of how it will be executed is explained herein.

A critical element of an electron gun is the thermionic cathode. The emission properties of any small portion of it may be viewed as the emitting surface of a 1-dimensional diode. The solution for this 1-dimensional diode model may be found in many references; and it is called the 3/2 power, or Child-Langmuir law. The simple solution is an elegant display of solving Poisson's equation, but it does have some assumptions that make it difficult to implement in a program that uses a particle-in-cell (PIC) approach.

To obtain a clearer understanding of the modeling difficulties, the actual process of the emission details should be considered first. The physical characteristics of the emission may be thought of in the following manner. At the specified cathode temperature, the electrons are emitted with a classic Maxwell-Boltzmann (M-B) energy distribution. As the anode potential is increased, the collected current increases and will eventually saturate. This condition is denoted as the "thermal limited regime"; See the figure below.

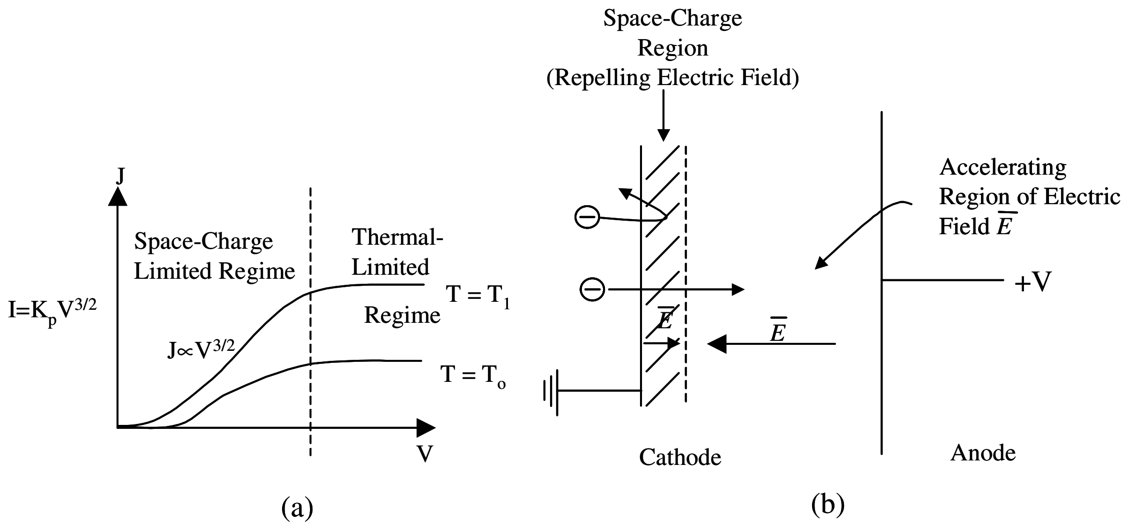

Figure 1 - (a) The J-V family of curves showing the space-charge-limited and the thermal-limited regimes. The parameter T is the Cathode temperature; $T_1 > T_0$. (b) Schematic of the 1-dimensional diode. Near the cathode is the dense space-charge layer with its resulting decelerating field.

As the anode potential is reduced, the collected current will drop, and the diode is in its 3/2 law regime, which is called the space charge limited region. In this case the reduced electric field at the cathode does not drain off the thermally emitted particles fast enough, and a space-charge layer forms adjacent to the cathode. The space-charge layer acts to repel some of the thermally emitted electrons back into the cathode surface. This has the effect of retarding the slower particles, and only allowing the more energetic ones to climb the potential hill due to the space charge layer, and then proceed on to the anode. Experimentally [1], [2] it is known that the energy distribution of the ensemble that overcomes the retarding space-charge field is still M-B, but with a lower effective temperature.

The space-charge layer causes a retarding electric field at the cathode surface, which decays rapidly to zero as one moves toward the anode. Beyond this point, the field is reversed in direction and begins to increase, and serves to accelerate the electrons toward the anode. The plane where the field is zero is also the point of the potential minimum (the virtual cathode). It is at this plane where the classic boundary conditions are applied. Physically, the electric field is zero there, and the potential is just a few tenths of a volt below zero, but the carrier velocity is not zero; it is of the order of the thermal speed. The electrons leave the cathode with velocities around 10^5 m/sec (their thermal speeds). Since they are generally accelerated up to speeds of 10^6 or 10^7 m/sec in typical applications, their initial speeds at the real and virtual cathodes may be neglected. The potential minimum is very close to the actual cathode, and it is only a few tenths of a volt below the grounded cathode; so at this plane, the electric field, potential, and velocity are set to zero. These boundary conditions yield a solution that is very close to measured results. One problem is the fact that the charge density must be infinite (since the velocity is zero) at the virtual cathode, since the current must be constant in a 1-dimensional problem. In a real diode, the charge density near the cathode is extremely large, but its value is not of particular importance; only the current-voltage relationship is relevant in device use.

To model a cathode with a PIC code, one could emit particles at the cathode with a M-B energy distribution and a net current equal to the measured saturated thermal current density. Then for any applied anode potential, the space-charge layer would form appropriately and the collected current should match that measured to within numerical and experimental error. Unfortunately this straight-forward recipe is impossible to implement. A large fraction of the thermally emitted electrons are repelled back into the surface and do not contribute to the collected current, and are wasteful of computer resources. Also, the maintenance of the space-charge layer would be an overbearing tax on the computation time. The total number of particles to use gets to be unreasonable, and the run time would be prohibitive. Finding a way, or several ways, to circumvent this dilemma constitutes the discussion of this memorandum.

Existing Electron Gun Software

While an exhaustive study of all available electron gun simulators has not been attempted, one of the most popular ones has been researched in some depth. The program is called EGUN, and was developed by Hermannsfeldt [3]. The program uses a “starting surface” which is several mesh cells away from the actual cathode. Current filaments are formed on the surface, and their values are varied until the net cathode current and anode voltage satisfy the 3/2 power law. That is the current is proportional to the 3/2 power of the anode voltage. The proportionality constant is the perveance of the gun, which is a function of the geometry of the electrodes. Notice this approach completely sidesteps the problem of forming the space-charge layer. In a real electron gun the electric field across the cathode surface varies due to the electrode geometry and applied potentials. This results in a variation of the space charge limited current across the surface, which is called variable cathode loading. This variation may change by 20%, with the largest emission density near the edge of the cathode, and the lowest emission density near the center. The program EGUN does calculate this variable loading phenomena. The electric field and potential are held to zero on the starting surface, and the iteration proceeds. The current filaments act on one another in a reasonable approximation, and magnetic focusing is incorporated. One level of approximation used on the initial EGUN calculation is the following. The documentation of the program dated in November 1979 (p. 85), says the program has difficulty determining the space charge on the beam axis. The calculated value is multiplied by the empirical factor of 5.5 to agree with some measured results (the details were not given). Notice the beam is “cold” (no thermal velocity), and thus noiseless. To simulate the noise in the beam, each filament is then subdivided into rays of varying strength and they are emitted at specified angles from the normal to the starting surface. These filaments are quickly curved back toward the direction of the normal, as no crossing of the filaments is allowed in the calculation. It has also been stated [4] that the program determines the perveance to within a few percent, but has a somewhat harder time predicting the beam diameter. Unfortunately, for our analysis effort, the beam diameter determination is critical. This basic scheme is used by others [5], and is also incorporated in MAFIA as a macro entitled SCLE (Space Charge Limited Emission).

Analysis Summary

With the physical concepts of the emission process in mind, the approach to model an electron gun using the program MAFIA is outlined here. The first step is to use the SCLE module to get the appropriate variation of cathode loading for the chosen electrode configuration. Next the cathode will be partitioned into segments (patches) where each segment is assumed to emit a uniform current density. Using this uniform current density in the patch, we will decompose it into the proper surface carrier density and velocity distribution to model a M-B distribution. Notice this step is emitting only those particles that have overcome the potential hill, (virtual cathode) and will be collected at the anode. The formation of the M-B distribution will be developed in the very long chapter entitled “Emission Modeling”.

MAFIA requires the specification (in a patch) of the number of emitted particles, their charge, mass, energy (speed), position, and emission directions; this must be kept in mind in the “Emission Modeling” section. Notice the cathode is at temperature T_1, with the appropriate M-B distribution for that temperature. At the voltage minimum (virtual cathode, and the location of the starting surface) the effective temperature is T_{eff}. We must determine T_{eff}, and the correct M-B distribution at this plane. A procedure is developed to provide this information.

The emitted particles (in a patch) are grouped into 14 discrete energy bins, and the variation in the number emitted as a function of angle from the normal is determined exactly. After much trial and error it was decided to always emit the same number of particles for every simulation. The variation in current for different conditions is handled by varying the charge of the emitted particles (called macroparticles). In a M-B distribution the number of particles in a given energy (speed) bin is dependent on that energy. By altering the charge and mass of the particles, we may use the same number of particles in each bin. The adjustments conserve both energy and momentum, and the change in mean-free-path is not severe. For example, we consider a low voltage diode later to get a feel for the magnitude of numbers involved in a calculation. If the diode is solved using electrons, their mean free path is about 5.89 microns. When we use our derived macroparticles, the path length is 15.67 microns. This variation will be assumed to be acceptable. The motivation for this plan is to keep each run to about the same time duration. From previous work it is known that using about 300,000 particles per run requires about 4-6 hours of computing time.

To verify the outlined scheme, a series of MAFIA computations for a known solution for a 1-dimensional diode will be carried out. The details of the known solution are in the chapter “Low Voltage Diode”.

Chapter 1 Emission Modeling

We know the energy distribution for the emitted particles is M-B both at the metallic cathode, and more importantly, at the virtual cathode (plane of the potential minimum). The program MAFIA requires the specification of: a) area of the emitting surface, b) the number of particles and their positions, c) the particle speed, and d) the particle direction. We may also specify the time step for successive bunch emissions. Given the temperature of a cathode T_1, and the material, we can use Figure 2.3-1 of Gewartowski [6] to determine the saturation emission current density J_o (Amps/cm^2). The value J_o is the limit in the thermal limited regime. (see [6] pages 42, 59, and 615). The sketch below indicates the typical J-V plot for a thermionic cathode.

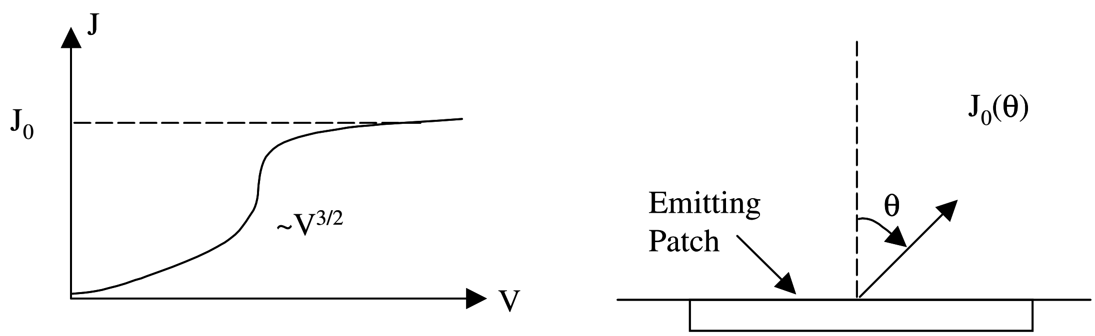

Figure 2 - The J-V curve for a single temperature, along with the $J_0(\theta)$ for a small emitting patch.

If we choose $T_1 = T = 1100^o\,K$, we can assume the typical value of $J_o = 0.5\ A/cm^2$. In the above sketch we have indicated an emitting patch and the current density in the direction θ from the normal. Let $n_p\,(\theta,\ \phi)$ and $v\,(\theta,\ \phi)$ be the number of particles emitted and their velocities in the $(\theta,\ \phi)$ direction, respectively. Here θ is the polar angle from the normal, and ϕ is the azimuthal measure. The average velocity, $<v>$, of an emitted particle (averaged over all directions) may be obtained from

$$\frac{1}{2}m\langle v\rangle^2 = 2W_T \tag{1-1}$$

where m is the mass of a particle, and $2W_T$ is their thermal energy. W_T is

$$W_T = \frac{kT}{|q|} = \frac{T}{11{,}600}eV$$

where k is Boltzmann's constant. For our example,

$$2W_T = 2kT = 2\left(1.38\times10^{-23}\,{}^{J}\!/\!{}_{\circ K}\right)\left(1100^\circ K\right)$$
$$= 3.036\times10^{-20}\,Joule$$

$$\therefore \frac{3.036\times10^{-20}\,J}{1.602\times10^{-19}\,{}^{J}\!/\!{}_{eV}} = 0.1895eV \quad \therefore W_T = .095eV$$

Then

$$\langle v\rangle = \sqrt{\frac{4W_T}{m}} = \left[\frac{6.072\times10^{-20}\,J}{9.108\times10^{-31}\,Kg}\right]^{1/2}$$

Hence

$$\langle v\rangle = 2.582\times10^5\,m/\sec$$

We arbitrarily choose ¼ (W_T) for energy of the slowest particle and 5.75 (W_T) for the energy of the fastest particle; then

$$v_{SLOW} = 1.291\times10^5\,m/\sec$$
$$v_{FAST} = 6.191\times10^5\,m/\sec$$

so we have a sense of the order of magnitudes with which we are dealing. Observe the ratio of fastest to slowest is only about 4.8.

Assume no ϕ dependence, then

$$J_0(\theta) = q\sum n_p(\theta)v(\theta) \tag{1-2}$$

where q = the particle charge.

From reference [7] we have:

N_R = total number of ions emitted per unit area per unit of time in the joint interval $du\ d\theta\ d\phi$ is

$$N_R = \frac{2h^2m^2}{\pi} u^3 e^{-hmu^2} \sin\theta \cos\theta\ du\ d\theta\ d\phi \tag{1-3}$$

where m = mass of ion (here just an electron), $h = 1/2kT$ (see pg 156), and u is a particular velocity. Note that the average energy of an emitted particle is $2kT$ (see eq. 1-1). This is in contrast to the value of $3kT/2$ for a particle in the classical M-B gas. This is due to the fact that the group that has escaped, must have enough energy to overcome the retarding binding force at the cathode surface. For the particles remaining in the metal, their average energy is $3kT/2$. See Appendix I for the derivation of this result. Thus, the mean kinetic energy (*KE*) in the metal may be expressed as 3/(4h). Integrate over ϕ;

$$N_R\left(u,\theta\right) = 4h^2m^2u^3e^{-hmu^2} \sin\theta \cos\theta\ du\ d\theta \tag{1-4}$$

Observe this is eq. (2.4-8) of Gewartowski [6]. In eq. (1-4), u ranges from 0 to ∞, but we know the major contribution is obtained during the interval

$$v_{SLOW} \leq u \leq v_{FAST}$$

Consider a group of particles with the same velocity (speed, call it u^1); then their distribution with θ is

$$N_R^1\left(u^1,\theta\right) = \left(const.\right)\cdot \sin\theta \cos\theta\ du^1 d\theta \tag{1-5}$$

Then the number per unit solid angle (U.S.A.) is

$$N_R^1\left(u^1,\theta\right)_{U.S.A.} = const \cdot \frac{\sin\theta \cos\theta}{2\pi \sin\theta\ d\theta} du^1 d\theta \tag{1-6}$$

$$N_R^1\left(u^1,\theta\right)_{U.S.A.} = const^1 \cdot \cos\theta\ du^1 \tag{1-7}$$

We interpret eq. (1-7) as stating "for a given particle speed, those with that speed are distributed as $\cos\theta$." The units of $N_R^1\left(u^1,\theta\right)$ are [particles/unit area]/[unit of time/U.S.A.] which equals [particle flux/U.S.A.]. Equation (1-7) is similar to the result of Gewartowski as follows. Start with his eq. (2.4-8)

$$dP(u,\theta) = \sin\theta\cos\theta\left(\frac{m}{kT}\right)^2 u^3 e^{-mu^2/kT}\,du\,d\theta \tag{1-8}$$

where $dP(u,\theta)$ is the probability that the emission velocity lies in the range u to u + du and makes an angle with the normal in the range θ to $\theta + d\theta$.

Integrate over all velocities (then the velocity dependence is removed).

$$\begin{aligned}\int_0^\infty dP(u,\theta)du &= \sin\theta\cos\theta(c)^2\int_o^\infty u^3 e^{-\frac{cu^2}{2}}du\\ &= \sin\theta\cos\theta(c)^2\left[\frac{1}{2\left(c/2\right)^2}\right] = 2\sin\theta\cos\theta\,d\theta\\ \therefore dP(\theta) &= 2\sin\theta\cos\theta\,d\theta\end{aligned} \tag{1-9}$$

Which is eq. (2.4-11) of Gewartowski. Now we say

$J(\theta)$ = [total emission current averaged over all directions] x $\dfrac{dP(\theta)}{U.S.A.}$

$$= J_o\frac{dP(\theta)}{U.S.A.} = J_o\frac{2\sin\theta\cos\theta\,d\theta}{2\pi\sin\theta\,d\theta}$$

= $\dfrac{J_o}{\pi}\cos\theta$ emitted current density/U.S.A., which is Gewartowski's eq. (2.4-12).

For a given T, we know the range of emission energies. The fastest .5% of particles will have energy about $6W_T$. The distribution of particles as a function of energy is

$$dN_E = \frac{2N_{TOT}}{E_T\sqrt{\pi}}\sqrt{\frac{E}{E_T}}e^{-E/E_T}dE \tag{1-10}$$

where $E_T = W_T$ and $E = W$. Notice the symbol for energy will be either W or E; we do this to conform to whichever author we are paralleling at a certain point in the analysis. Let $\rho_\eta = dN_\eta / d\eta$ where $\eta = E / E_T$.

N_{TOT} = particle density (particles/m^3). We will subdivide the range of energy into B energy bins. For now B is arbitrary. The velocity of a particle in bin_j is

$$v_j = \sqrt{\frac{2E_j}{m}} \tag{1-11}$$

Let n_j be the particle density of those with energy E_j. The current from a patch is

$$I_{PATCH} = J_o A_{PATCH} \tag{1-12}$$

where A_{PATCH} is the patch area. Write J_o in $Amps/m^2$ and A_{PATCH} in m^2.

$$I_{PATCH} = J_o A_{PATCH} = qA_{PATCH}\left(n_1 v_1 + n_2 v_2 + \cdots + n_B v_B\right) \tag{1-13}$$

where n_j = density of particles in bin_j j = 1,2…,B

v_j = velocity of particles in bin_j

We will show

$$\begin{aligned} n_1 &= \propto_1 N_{TOT} \\ n_2 &= \propto_2 N_{TOT} \\ &\vdots \\ n_B &= \propto_B N_{TOT} \end{aligned} \tag{1-13a}$$

Where the $\propto_j$ are known constants.

Then

$$I_{PATCH} = qA_{PATCH} N_{TOT}\left(\propto_1 v_1 + \propto_2 v_2 + \cdots + \propto_B v_B\right)$$

but the v_j and $\propto_j$ are known, so

$$N_{TOT}\left(\frac{particles}{m^3}\right) = \frac{I_{PATCH}}{qA_{PATCH}\sum \propto_j v_j} \quad j = 1,\ldots,B \tag{1-14}$$

Once N_{TOT} is found, n_j are computed from eq. (1-13a).

From Millman and Seely [8], their section 5-15, we have

$$KE_{NORMAL} = 2KE_{TANGENTIAL} \tag{1-15}$$

which states the *KE* normal to the surface is twice the tangential value, when calculated over the ensemble. See Appendix I for its verification. Let n_{ji} represent the particle density in energy bin_j and emitted at angle θ_i. Let v_j be the velocity of particles in bin_j. For calculation purposes we will choose 4 energy bins (j = 1 to 4) and 5 (i = 0 to 4) emission angles. The sketch below illustrates the model.

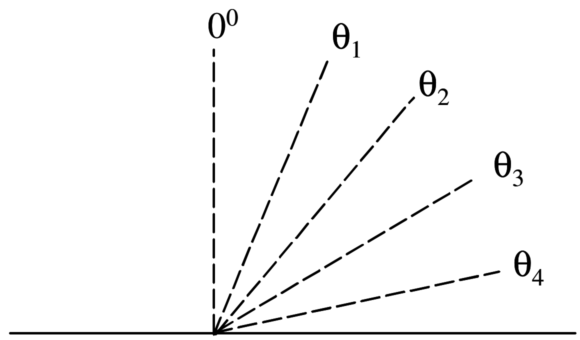

Figure 3 - The case for emission at 5 specific angles from a small patch. Emission occurs at 0^0 as well as θ_1 through θ_4.

Eq. (1-15) becomes

$$\frac{1}{2}\left\{\left(n_{10}v_1^2 + n_{20}v_2^2 + n_{30}v_3^2 + n_{40}v_4^2\right) + \left(n_{11}v_1^2 + n_{21}v_2^2 + n_{31}v_3^2 + n_{41}v_4^2\right)\cos^2\theta_1 + \right.$$
$$\left(n_{12}v_1^2 + n_{22}v_2^2 + n_{32}v_3^2 + n_{42}v_4^2\right)\cos^2\theta_2 + \left(n_{13}v_1^2 + n_{23}v_2^2 + n_{33}v_3^2 + n_{43}v_4^2\right)\cos^2\theta_3 +$$
$$\left.\left(n_{14}v_1^2 + n_{24}v_2^2 + n_{34}v_3^2 + n_{44}v_4^2\right)\cos^2\theta_4\right\} = RHS$$

Where *RHS* means the right hand side of eq. (1-15).

But we know the velocities at the various angles are related by constants, i.e.

$$v_2 = \kappa_2 v_1, \quad v_3 = \kappa_3 v_1, \quad v_4 = \kappa_4 v_1$$

$$
\begin{array}{ll}
n_{11} = n_{10}\cos\theta_1 & n_{21} = n_{20}\cos\theta_1 \\
n_{12} = n_{10}\cos\theta_2 & n_{22} = n_{20}\cos\theta_2 \\
n_{13} = n_{10}\cos\theta_3 & n_{23} = n_{20}\cos\theta_3 \\
n_{14} = n_{10}\cos\theta_4 & n_{24} = n_{20}\cos\theta_4 \\
n_{31} = n_{30}\cos\theta_1 & n_{41} = n_{40}\cos\theta_1 \\
n_{32} = n_{30}\cos\theta_2 & n_{42} = n_{40}\cos\theta_2 \\
n_{33} = n_{30}\cos\theta_3 & n_{43} = n_{40}\cos\theta_3 \\
n_{34} = n_{30}\cos\theta_4 & n_{44} = n_{40}\cos\theta_4
\end{array}
$$

So

$$
\begin{aligned}
&\frac{1}{2}\Big\{n_{10}v_1^2 + n_{20}\left(\kappa_2^2\right)v_1^2 + n_{30}\left(\kappa_3^2\right)v_1^2 + n_{40}\left(\kappa_4^2\right)v_1^2 \\
&+\left(n_{10}v_1^2 + n_{20}\kappa_2^2 v_1^2 + n_{30}\kappa_3^2 v_1^2 + n_{40}\kappa_4^2 v_1^2\right)\cos^3\theta_1 \\
&+\left(n_{10}v_1^2 + n_{20}\kappa_2^2 v_1^2 + n_{30}\kappa_3^2 v_1^2 + n_{40}\kappa_4^2 v_1^2\right)\cos^3\theta_2 + (\quad)\cos^3\theta_3 \\
&+(\quad)\cos^3\theta_4\Big\} = \frac{1}{2}v_1^2 X\left\{1+\cos^3\theta_1+\cos^3\theta_2+\cos^3\theta_3+\cos^3\theta_4\right\}
\end{aligned} \tag{1-16}
$$

Where

$$
X = n_{10} + n_{20}\kappa_2^2 + n_{30}\kappa_3^2 + n_{40}\kappa_4^2 \tag{1-16a}
$$

Now the right hand side (*RHS*) of eq. (1-15) is

$$
\begin{aligned}
&\Big\{\left(n_{11}v_1^2 + n_{21}v_2^2 + n_{31}v_3^2 + n_{41}v_4^2\right)\sin^2\theta_1 + \left(n_{12}v_1^2 + n_{22}v_2^2 + n_{32}v_3^2 + n_{42}v_4^2\right)\sin^2\theta_2 \\
&+\left(n_{13}v_1^2 + n_{23}v_2^2 + n_{33}v_3^2 + n_{43}v_4^2\right)\sin^2\theta_3 + \left(n_{14}v_1^2 + n_{24}v_2^2 + n_{34}v_3^2 + n_{44}v_4^2\right)\sin^2\theta_4\Big\}
\end{aligned}
$$

Or

$$
\begin{aligned}
&\Big\{\left(n_{10}v_1^2 + n_{20}\kappa_2^2 v_1^2 + n_{30}\kappa_3^2 v_1^2 + n_{40}\kappa_4^2 v_1^2\right)\cos\theta_1\sin^2\theta_1 + \\
&(\quad)\cos\theta_2\sin^2\theta_2 + (\quad)\cos\theta_3\sin^2\theta_3 + (\quad)\cos\theta_4\sin^2\theta_4\Big\} \\
&= v_1^2\left\{X\left(\cos\theta_1\sin^2\theta_1 + \cos\theta_2\sin^2\theta_2 + \cos\theta_3\sin^2\theta_3 + \cos\theta_4\sin^2\theta_4\right)\right\}
\end{aligned}
$$

Then

$$\tfrac{1}{2}v_1^2 X\{1+\cos^3\theta_1+\cos^3\theta_2+\cos^3\theta_3+\cos^3\theta_4\} = v_1^2 X(\cos\theta_1\sin^2\theta_1 + \cos\theta_2\sin^2\theta_2+\cos\theta_3\sin^2\theta_3+\cos\theta_4\sin^2\theta_4)$$

Then

$$1+\cos^3\theta_1+\cos^3\theta_2+\cos^3\theta_3+\cos^3\theta_4 = 2\cos\theta_1\sin^2\theta_1+\cdots$$

Or

$$\begin{aligned}&\tfrac{1}{2}+\left(\tfrac{1}{2}\cos^3\theta_1-\cos\theta_1\sin^2\theta_1\right)+\left(\tfrac{1}{2}\cos^3\theta_2-\cos\theta_2\sin^2\theta_2\right)\\&+\left(\tfrac{1}{2}\cos^3\theta_3-\cos\theta_3\sin^2\theta_3\right)+\left(\tfrac{1}{2}\cos^3\theta_4-\cos\theta_4\sin^2\theta_4\right)=0\end{aligned} \tag{1-17}$$

Now define

$$f(\theta)=\tfrac{1}{2}\cos^3\theta-\cos\theta\sin^2\theta \tag{1-18}$$

which is sketched below.

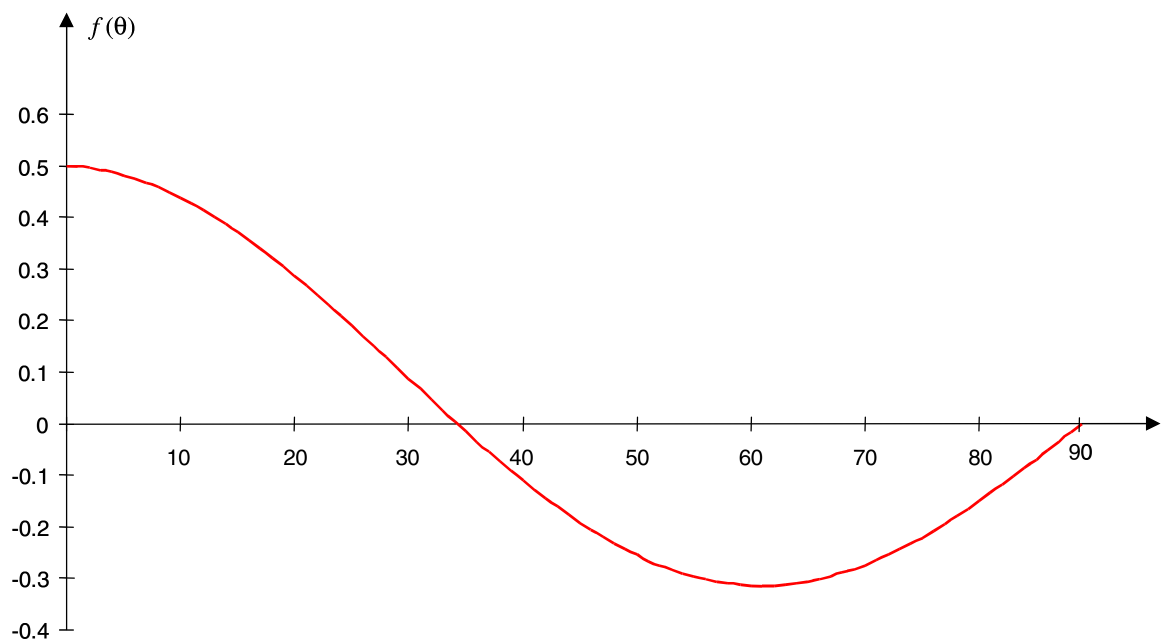

Figure 4 - A sketch of the function *f*(θ) in eq. (1-18).

Rearrange eq. (1-17) as

$$f(\theta_1) + f(\theta_2) + f(\theta_3) + f(\theta_4) = -\tfrac{1}{2} \qquad \text{(1-17a)}$$

So we must choose the $\theta_1, \theta_2, \theta_3, \theta_4$ to satisfy eq. (1-17a)

$\theta°$	$f(\theta)$	$\theta°$	$f(\theta)$	$\theta°$	$f(\theta)$	$\theta°$	$f(\theta)$
1	.499	22	.268	43	-.145	64	-.312
2	.498	23	.249	44	-.161	65	-.309
3	.495	24	.230	45	-.177	66	-.306
4	.492	25	.210	46	-.192	67	-.301
5	.487	26	.190	47	-.206	68	-.296
6	.481	27	.170	48	-.220	69	-.289
7	.474	28	.150	49	-.232	70	-.282
8	.466	29	.129	50	-.244	71	-.274
9	.458	30	.108	51	-.255	72	-.265
10	.448	31	.088	52	-.266	73	-.255
11	.437	32	.067	53	-.275	74	-.244
12	.426	33	.046	54	-.283	75	-.233
13	.413	34	.026	55	-.291	76	-.221
14	.400	35	.0053	56	-.297	77	-.208
15	.386	36	-.015	57	-.302	78	-.194
16	.371	37	-.035	58	-.307	79	-.180
17	.356	38	-.054	59	-.310	80	-.166
18	.339	39	-.073	60	-.312	81	-.151
19	.322	40	-.092	61	-.314	82	-.135
20	.305	41	-.110	62	-.314	83	-.119
21	.287	42	-.128	63	-.314	84	-.103

85	-.086	87	-.052	89	-.017		
86	-.069	88	-.035	90	0.0		

The following 17 sets of angles that satisfy eq. (1-17a) will be listed for reference purposes. Notice that every angle from 1 to 90 is used. The sets were chosen randomly.

Set	Angles
1	10,20,30,40,50,52,60,70,80
2	5,15,25,40,45,50,55,60,70,81,88
3	15,20,30,43,48,53,58,68,87
4	7,14,28,35,36,37,40,60,65,70,75,80,85
5	2,12,31,42,47,51,56,62,74,86
6	11,19,63,67,73,77,79
7	3,8,54,57,63,67,72
8	1,59,66,71,84
9	4,61,73,76,77
10	23,64,69,82,89
11	22,24,26,41,44,46,49,78,79,83
12	6,18,34,38,39,45,46,57,61,75
13	9,52,64,73,77,85,88
14	13,32,68,71,73,81
15	16,33,48,64,74,84,88
16	17,29,42,62,68,76,89
17	21,27,34,44,56,67,77

Some 4 angle sets are: (notice their sum should be -0.5)

15,60,66,71 their sum = -.506

20,48,57,70 their sum = -.499

25,62,75,80 their sum = -.503

30,68,77,84 their sum = -.499

Recall

$$\left(n_{10} + n_{11} + n_{12} + n_{13} + n_{14}\right) = n_1$$

Or

$$n_{10}\left(1 + \cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4\right) = n_1$$

$$\therefore n_{10} = \frac{n_1}{1 + \sum \cos\theta_i} \qquad (1\text{-}19)$$

And n_{20}, n_{30}, n_{40} can be expressed similarly. Thus all 16 bunches n_{ji}, i,j=1,2,3,4 are determined. For each angle set, we will distribute them uniformly in phi (the azimuthal angle).

Now we calculate the n_j. Recall from eq. (1-13a)

$$n_1 = \propto_1 N_{TOT}$$

$$n_j = \propto_j N_{TOT}$$

The energy distribution function is shown in the plot below

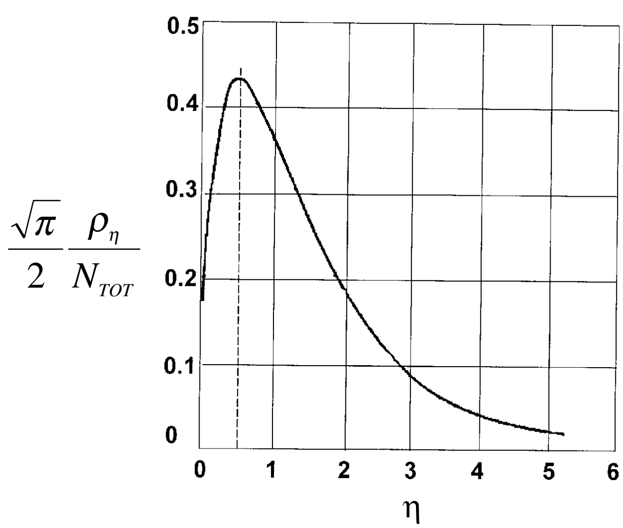

Figure 5 - The Maxwell-Boltzmann energy distribution function plotted versus the normalized variable η=E/E_T.

Consider a small patch which emits N_{TOT} particles over some specified time interval. Subdivide the emitted particles into 14 groups (now we will use 14 energy bins), with the particles in each bin having the same energy, (and thus speed). By graphically making the partitions, we may calculate the number of particles in bin_j

$$n_j = \frac{2N_{TOT}}{\sqrt{\pi}} \int_{bin_j} \sqrt{\eta} e^{-\eta} d\eta \tag{1-20}$$

Evaluating eq. (1-20) in general terms gives

$$n = \int_{\eta_1}^{\eta_2} \sqrt{\eta} e^{-\eta} d\eta$$

Where we have suppressed the coefficient $2N_{TOT} / \sqrt{\pi}$

Let

$$x = \sqrt{\eta}, \qquad x^2 = \eta, \qquad 2xdx = d\eta$$

So

$$n = 2\int_{x_1}^{x_2} x^2 e^{-x^2} dx \tag{1-21}$$

Let

$$\begin{aligned} f &= e^{-x^2} \\ f^1 &= -2xf \\ f^{11} &= -2xf^1 - 2f = 4x^2 f - 2f \end{aligned}$$

Or

$$x^2 f = \tfrac{1}{4}\left(f^{11} + 2f\right)$$

Then

$$2\int_{x_1}^{x_2} x^2 e^{-x^2}\, dx = \tfrac{1}{2}\int_{x_1}^{x_2} \left(f^{11} + 2f\right) dx$$

$$= \tfrac{1}{2} f^{1}\Big|_{x_1}^{x_2} + \int_{x_1}^{x_2} e^{-x^2}\, dx = \tfrac{1}{2}\left[f^{1}(x_2) - f^{1}(x_1)\right] + \int_{0}^{x_2} e^{-x^2}\, dx$$

$$- \int_{0}^{x_1} e^{-x^2}\, dx = \tfrac{1}{2}\left[2x_1 f(x_1) - 2x_2 f(x_2)\right] + \frac{\sqrt{\pi}}{2}\left[erf(x_2) - erf(x_1)\right]$$

So

$$n = x_1 f(x_1) - x_2 f(x_2) + \frac{\sqrt{\pi}}{2}\left[erf(x_2) - erf(x_1)\right]$$

Thus

$$n_j = \frac{2N_{TOT}}{\sqrt{\pi}} n = N_{TOT}\left\{\frac{2}{\sqrt{\pi}}\left(x_1 e^{-x_1^2} - x_2 e^{-x_2^2}\right) + erf(x_2) - erf(x_1)\right\} \qquad (1\text{-}22)$$

Our values are:

η	x	e^{-x^2}	xe^{-x^2}	$erf(x)$
0	0	1	0	0
.5	.70711	.60653	.42888	.68260
1	1	.36788	.36788	.84270
1.5	1.2247	.22313	.27327	.91679
2.0	1.4142	.13534	.1914	.95449
2.5	1.5811	.082085	.12978	.97464
3.0	1.7321	.049787	.086236	.98569
3.5	1.8708	.030197	.056493	.99185
4.0	2.0	.018316	.036632	.99532
4.5	2.1213	.011109	.023566	.9973
5.0	2.2361	.0067379	.015067	.99843
5.5	2.3452	.0040868	.0095844	.99909
6.0	2.4495	.0024788	.0060718	.99947
6.5	2.5495	.0015034	.0038329	.99969
7.0	2.6458	.00091188	.0024127	.99982

Then

$$n_1 = N_{TOT}\left\{\frac{2}{\sqrt{\pi}}\left(-.4288\right)+.6826\right\} = .19866\,N_{TOT}$$

$$n_2 = N_{TOT}\left\{\qquad\right\} = .2289\,N_{TOT}$$

$$n_3 = .18085 N_{TOT}$$

$$n_4 = .13008 N_{TOT}$$

$$n_5 = .089682 N_{TOT}$$

$$n_6 = .060185 N_{TOT}$$

$$n_7 = .039722 N_{TOT}$$

$$n_8 = .025881 N_{TOT}$$

$$n_9 = .016724 N_{TOT}$$

$$n_{10} = .01072 N_{TOT}$$

$$n_{11} = 6.8466\times10^{-3}\,N_{TOT}$$

$$n_{12} = 4.3436\times10^{-3}\,N_{TOT}$$

$$n_{13} = 2.7464\times10^{-3}\,N_{TOT}$$

$$n_{14} = 1.7326\times10^{-3}\,N_{TOT}$$

As a check,

$$\sum_{j=1}^{14} n_j = .9971 N_{TOT} \tag{1-23}$$

Thus the final table is:

Bin	$\bar{\eta}_j$	n_j/N_{TOT}
1	.25	.19866
2	.75	.22893
3	1.25	.18085
4	1.75	.13008
5	2.25	.089682

6	2.75	.060185
7	3.25	.039722
8	3.75	.025881
9	4.25	.016724
10	4.75	.01072
11	5.25	6.8466x10^{-3}
12	5.75	4.343x10^{-3}
13	6.25	2.7464x10^{-3}
14	6.75	1.7326x10^{-3}

Where $\bar{\eta}_j$ is the normalized energy parameter at the center of the partition. The emission velocity is developed as follows.

$$\tfrac{1}{2} m v_j^2 = qE_j; \qquad write\ E_j = 2E_T \bar{\eta}_j$$

Where $E_T = T/11{,}600$; the factor of 2 is to account for those that have escaped.

$$\tfrac{1}{2} m v_j^2 = 2\bar{\eta}_j \left(\frac{T}{11{,}600} \right) = 2\bar{\eta}_j kT \tag{1-24}$$

or

$$v_j = \sqrt{\frac{4kT}{m}\bar{\eta}_j} = \left[\frac{4(1.38\times10^{-23})}{9.108\times10^{-31}} \right]^{1/2} \sqrt{\bar{\eta}_j T} \tag{1-25}$$
$$= 7.785\times10^3 \sqrt{\bar{\eta}_j T}\ m/\sec$$

For our case, T =1100° K, thus $v_j = 2.582\times10^5 \sqrt{\bar{\eta}_j}$ (1-26)

Bin	$\bar{\eta}_j$	n_j/N_{TOT}	v_j (m/sec)
1	.25	.19866	1.29×10^5
2	.75	.22893	2.2361×10^5
3	1.25	.18085	2.8868×10^5
4	1.75	.13008	3.4157×10^5
5	2.25	.089682	3.873 “
6	2.75	.060185	4.282 “
7	3.25	.039722	4.655 “
8	3.75	.025881	5.0×10^5
9	4.25	.016724	5.32 “
10	4.75	.01072	5.627 “
11	5.25	6.8466×10^{-3}	5.92 “
12	5.75	4.3436×10^{-3}	6.19 “
13	6.25	2.7464×10^{-3}	6.46 “
14	6.75	1.7326×10^{-3}	6.71×10^5

Note $v_{FAST} / v_{SLOW} \doteq 5.2$

Gewartowski shows

$$J(\theta) = \frac{J_0}{\pi} \cos\theta \qquad \frac{Amps/cm^2}{U.S.A} \tag{1-27}$$

Assume all particles are identical, then

$$J(\theta) = q \sum \left[n_1(\theta,\phi) v_1 + n_2(\theta,\phi) v_2 + \cdots + n_{14}(\theta,\phi) v_{14} \right] \tag{1-28}$$

Where $n_1\ (\theta,\ \phi)$ = the number of particles in direction $(\theta,\ \phi)$ with speed v_1

Then

$$\int_\phi \int_\theta n_1(\theta,\phi) d\theta d\phi = n_1 = \kappa_1 N_{TOT}$$

In general

$$\int_{\phi}\int_{\theta} n_j(\theta,\phi)d\theta d\phi = n_j = \kappa_j N_{TOT} \tag{1-29}$$

It is shown in Richardson [7] that for a fixed velocity, the angular distribution is uniform in ϕ and cosine in theta (θ). Thus

$$n_j(\theta,\phi) = A_j \cos\theta \tag{1-30}$$

Then

$$\begin{aligned} \kappa_j N_{TOT} &= \int_{\theta=0}^{\pi/2} (2\pi) A_j \cos\theta \, d\theta = 2\pi A_j \\ \therefore A_j &= \frac{\kappa_j N_{TOT}}{2\pi} \end{aligned} \tag{1-31}$$

Observe eq. (1-30) satisfies a fundamental constraint; namely

$$KE_{normal} = 2 \times KE_{\tan gential} \tag{1-32}$$

For a given velocity

$$\begin{aligned} KE_{normal} &= \frac{m}{2}\int_{o}^{\pi/2} A_j \cos\theta \left(v_j \cos\theta\right)^2 d\theta = \frac{mA_j v_j^2}{2}\int_0^{\pi/2} \cos^3\theta d\theta \\ &= \frac{mA_j v_j^2}{2}\left(2/3\right) \end{aligned} \tag{1-33}$$

$$KE_{\tan gential} = \frac{mA_j v_j^2}{2}\int_0^{\pi/2} \cos\theta \sin^2\theta d\theta = \frac{mA_j v_j^2}{2}\left(1/3\right) \tag{1-34}$$

Observe eq. (1- 33) is twice eq. (1- 34), which validates the procedures used. Now combining eqns. (1-27) and (1-28)

$$J(\theta) = \frac{J_o}{\pi}\cos\theta = q\left\{\left(A_1\cos\theta\right)v_1 + \left(A_2\cos\theta\right)v_2 + \cdots + \left(A_{14}\cos\theta\right)v_{14}\right\}$$

Or

$$\frac{J_o}{\pi} = \frac{q}{2\pi}\left\{\kappa_1 N_{TOT} v_1 + \kappa_2 N_{TOT} v_2 + \cdots + \kappa_{14} N_{TOT} v_{14}\right\}$$
$$\therefore J_o = \frac{qN_{TOT}}{2}\sum_{j=1}^{14}\kappa_j v_j \tag{1-35}$$

Now we can determine N_{TOT} given J_o and T.

A typical example might use $J_o = 5\ Amp/cm^2 = 5 \text{ x } 10^4\ Amp/m^2$

Evaluating $\sum_1^{14}\kappa_j v_j$

$\kappa_1 v_1 = 2.565\times10^4$

$\kappa_2 v_2 = 5.119\times10^4$

$\kappa_3 v_3 = 5.221\times10^4$

$\kappa_4 v_4 = 4.443\times10^4$

$\kappa_5 v_5 = 3.473\times10^4$

$\kappa_6 v_6 = 2.577\times10^4$

$\kappa_7 v_7 = 1.849\times10^4$

$\kappa_8 v_8 = 1.294\times10^4$

$\kappa_9 v_9 = 8.902\times10^3$

$\kappa_{10} v_{10} = 6.032\times10^3$

$\kappa_{11} v_{11} = 4.051\times10^3$

$\kappa_{12} v_{12} = 2.689\times10^3$

$\kappa_{13} v_{13} = 1.773\times10^3$

$\kappa_{14} v_{14} = 1.162\times10^3$

$\sum_{j=1}^{14}\kappa_j v_j = 2.9\times10^5$

From eq. (1-35)

$$N_{TOT} = \frac{2J_o}{q}\frac{1}{\sum\kappa_j v_j}$$
$$= 2.15\times10^{18}\ {}^{e}\!/\!_{m^3}$$

$$A_1 = 6.798\times10^{16} \qquad A_8 = 8.856\times10^{15}$$
$$A_2 = 7.834\times10^{16} \qquad A_9 = 5.723\times10^{15}$$
$$A_3 = 6.188\times10^{16} \qquad A_{10} = 3.668\times10^{15}$$
$$A_4 = 4.451\times10^{16} \qquad A_{11} = 2.343\times10^{15}$$
$$A_5 = 3.069\times10^{16} \qquad A_{12} = 1.486\times10^{15}$$
$$A_6 = 2.059\times10^{16} \qquad A_{13} = 9.398\times10^{14}$$
$$A_7 = 1.359\times10^{16} \qquad A_{14} = 5.929\times10^{14}$$

While the above is without approximation (except for the finite number of partitions), the numbers are too large for MAFIA. There are many constraints on the emitted particles when using the program; some of them are: There is no easy way to incorporate the cosine distribution with θ. Instead, we'll use a parabolic distribution that closely matches a cosine. The number of particles/bin cannot be too large, or the run time will become prohibitive. The number of patches may be large, especially when modeling near the edge of the cathode. The question as to whether one should use systematic emission versus random emission is a very perplexing problem. The systematic case develops lower computation noise, but a random process seems to be what one needs. However, how does one produce such a condition, yet guarantee the emission is theoretically correct? Another constraint is the use of 4 to 5 particles per mesh cell to obtain good results. These along with other conditions will be explored and overcome in the succeeding work.

Chapter 2 Low Voltage Diode

To obtain a clearer perspective of the magnitudes of the parameters for a thermionic cathode, an analytical study of a typical 1-dimensional diode was undertaken. The following example is close to that given by Beck [9], pg. 173 (the example has some errors). The diode has plates with surface area of 0.2 cm^2 and separated the distance *d*, as shown below.

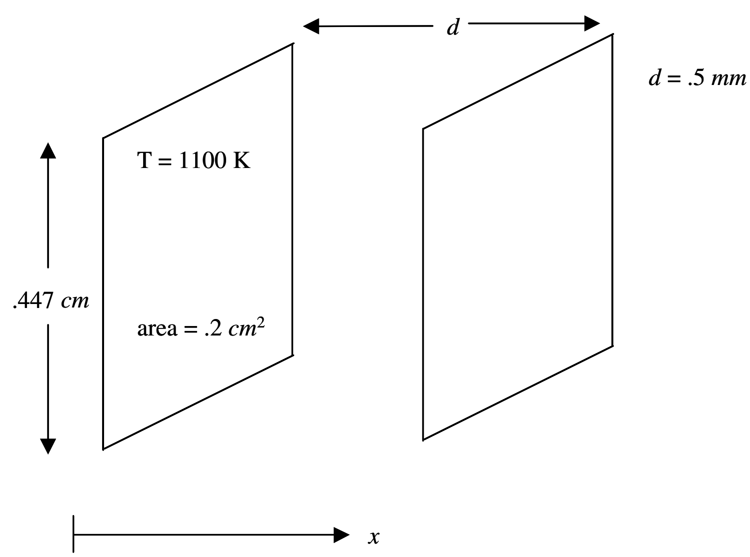

Figure 6 - The geometry of the 1-dimensional diode.

The current drawn is 1.0 mA, the current density is 5 x 10^{-3} A/cm^2 , and the thermal limited value J_o is 5.0 A/cm^2. At the cathode the parameter η is

$$\eta \equiv \eta_c = \ln(J_o / J) = \ln 10^3 = 6.9078$$

From the table on page 527 of [9], we see this corresponds to the parameter -ξ_c = 2.509. Where ξ is defined by

$$\xi = 9.174 \times 10^5 T^{-3/4} \sqrt{J}\left(x - x_m\right) \tag{2-1}$$

where x, x_m are in cm, J is in A/cm^2, and T is in Kelvins. The voltage minimum is located at x_m. This equation is taken from the original paper by Langmuir [10]. At the cathode, $\xi = \xi_c$ and $x = 0$. Then

$$-2.509 = -3.3963 \times 10^2 x_m$$

or

$$x_m = 7.4 \times 10^{-5} m = 0.074 mm$$

Then the voltage at the minimum is

$$V_m = -\frac{1100}{11{,}600} \ln 10^3 = -0.65471 volts$$

At the anode

$$\xi_a = 3.396 \times 10^2 (.05 - .0074) = 14.468$$

From the table in [9] we read $\eta_a = 19.27$

Now

$$\eta = \frac{11{,}606}{T}(V - V_m)$$

$$19.27 = \frac{11{,}606}{1100}(V_a + .65471)$$

$$\therefore V_a = 1.172\, volts$$

Where V_a is the required anode voltage.

With this information we may generate a plot of the potential through the diode versus x. The table is:

$x(cm)$	ξ	η	$V(volts)$
0	-2.509	6.9078	0
.000625	-2.301	3.5	-.323
.00125	-2.089	2.286	-.438
.0025	-1.6642	1.12	-.549

.005	-.8151	.2	-.636
.006	-.4755	.065	-.649
.007	-.1359	.0045	-.6543
.008	+.204	.001	-.6546
.009	+.543	.067	-.648
.010	+.883	.17	-.6386
.020	+4.279	2.8	-.389
.030	+7.676	7.2	+.028
.040	+11.07	12.7	+.549
.050	+14.47	19.27	+1.172

The plot of the voltage is shown below.

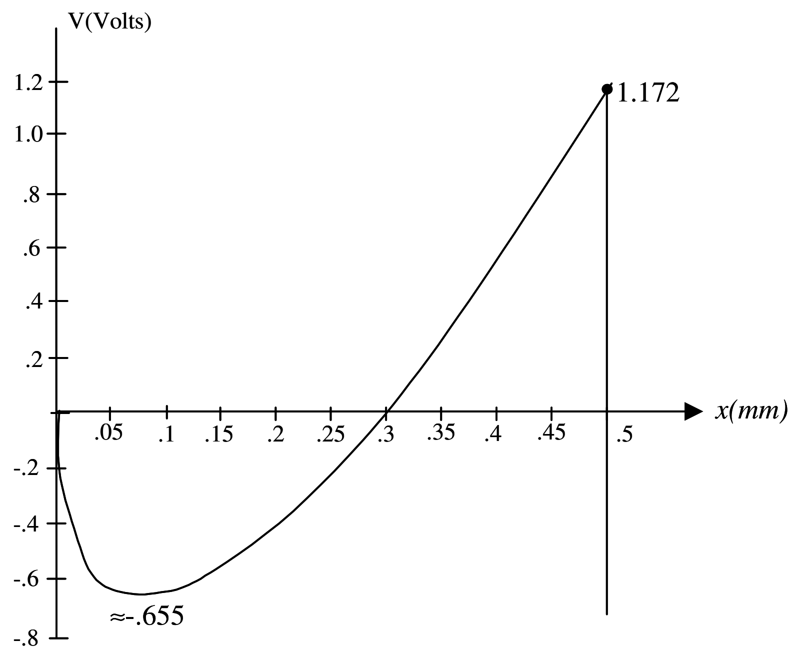

Figure 7 - A sketch of the voltage V(x) through the diode. The value at the minimum position (x_m) is about -0.655 volts.

We could numerically differentiate *V(x)* to obtain the electric field, $\overline{E}$ but to check for consistency between various treatments of the 1-dimensional diode, [11], we will use an analytic formula for $\overline{E}$ from [9]. For $x < x_m$ the field is given by

$$\overline{E}^2 = K\left[e^\eta - 1 + e^\eta erf\sqrt{\eta} - 2\left(\eta/\pi\right)^{1/2}\right]$$

For $x > x_m$, we have

$$\overline{E}^2 = K\left[e^\eta - 1 - e^\eta erf\sqrt{\eta} + 2\left(\eta/\pi\right)^{1/2}\right]$$

Where $\overline{E}$ is in *V/cm*, K = 6250J $\sqrt{T}$ = 1.0364 x 10^3 V^2/cm^2. The table is:

$x(cm)$	η	$\sqrt{\eta}$	$erf(\sqrt{\eta})$	$\overline{E}(V/cm)$
0	6.9078	2.6283	.9998	1.4382x10^3
.000625	3.5	1.8708	.9918	255
.00125	2.286	1.512	.967	131.3
.0025	1.12	1.058	.865	60.4
.005	.2	.447	.466	17.17
.006	.065	.255	.281	9.07
.007	.0045	.067	.075	10^{-4}
.010	.17	.4123	.438	-11.67
.030	7.2	2.683	.99985	-48.06
.040	12.7	3.56	1	-55.96
.050	19.27	4.3898	1	-64

The first and last $\overline{E}$-field entries are those at the cathode $\overline{E_c}$ and anode $\overline{E}_a$. The magnitude of the field at the anode is 64 *V/cm*, and from the graph of the potential, the slope at the anode is about 1 volt over .015 *cm*, or 66.7 *V/cm*. The agreement is well within tolerance.

Next we may determine the charge density $\rho(x)$ as follows. For convenience we first determine it in the accelerating part of the field;

For $x > x_m$

$$\overline{E}^2 = K\left[e^{\eta} - 1 - e^{\eta} erf\sqrt{\eta} + 2\left(\eta/\pi\right)^{1/2}\right]$$

Let $f_1(\eta) = e^{\eta} erf\left(\sqrt{\eta}\right)$

$$f_1^1(\eta) = e^{\eta}\left(\frac{2}{\sqrt{\pi}} e^{-\eta}\right) + erf\sqrt{\eta}\ e^{\eta} = \frac{2}{\sqrt{\pi}} + e^{\eta} erf\sqrt{\eta}$$

$$f(\eta) = e^{\eta} - 1 - f_1(\eta) + \frac{2}{\sqrt{\pi}}\eta^{1/2}$$

$$f^1(\eta) = e^{\eta} - f_1^1(\eta) + \frac{2}{\sqrt{\pi}}\frac{1}{2}\frac{1}{\sqrt{\eta}}$$

$$= e^{\eta} - \frac{2}{\sqrt{\pi}} - e^{\eta} erf\sqrt{\eta} + \frac{1}{\sqrt{\pi\eta}}$$

$$2\overline{E}\frac{d\overline{E}}{dx} = Kf^1(\eta)\frac{d\eta}{dx}$$

But

$$\frac{d\eta}{dx} = \frac{d\eta}{dV}\frac{dV}{dx} = -\overline{E}\frac{d\eta}{dV}$$

$$2\overline{E}\frac{d\overline{E}}{dx} = Kf^1(\eta)\left[-\overline{E}\frac{d\eta}{dV}\right]$$

$$\therefore 2\frac{\rho}{\varepsilon_o} = -K\left[e^{\eta} - \frac{2}{\sqrt{\pi}} - e^{+\eta} erf\sqrt{\eta} + \frac{1}{\sqrt{\pi\eta}}\right]\frac{d\eta}{dV}$$

where ε_o is the permittivity of free space.

Or

$$\rho(x) = -\frac{K\varepsilon_o}{2}\left[e^{\eta} - \frac{2}{\sqrt{\pi}} - e^{\eta} erf\sqrt{\eta} + \frac{1}{\sqrt{\pi\eta}}\right]\frac{d\eta}{dV}$$

$$= -\frac{K\varepsilon_o}{2}\left[e^{\eta} - \frac{2}{\sqrt{\pi}} - e^{\eta} erf\sqrt{\eta} + \frac{1}{\sqrt{\pi\eta}}\right]\left(\frac{11606}{1100}\right)$$

Note

$$\rho(x) = -\frac{K\varepsilon_o}{2}\left|f^{1}(\eta)\right|\left(\frac{11606}{1100}\right)$$

Where

$$K = 1.0364\times10^{3}\left(V/_{cm}\right)^{2} = 1.0364\times10^{7}\ V^{2}/_{m^{2}}$$

$$\therefore \rho(x) = -4.8409\times10^{-4}\left|f^{1}(\eta)\right| coul/_{m^{3}}$$

$$\xi = 3.3962\times10^{2}\left(x - x_m\right)$$

Our table is below.

$x(cm)$	ξ	η	$\sqrt{\eta}$	$erf\sqrt{\eta}$	$\left\|f^{1}(\eta)\right\|$	$\rho(c/m^{3})$
.0075	.03396	.00027	1.64E-2	--	--	--
.0076	.0679	.001	3.16E-2	.338	17.38	$-84.1\text{x}10^{-4}$
.008	.2038	.0096	9.8E-2	--	--	$-4.59\text{x}10^{-4}$
.009	.5434	.067	.259	.284	1.0993	$-5.32\text{x}10^{-4}$
.010	.883	.17	.412	.438	.906	$-4.39\text{x}10^{-4}$
.020	4.28	2.8	1.67	.982	.49522	$-2.4\text{x}10^{-4}$
.030	7.68	7.2	2.68	.99985	--	--
.040	11.07	12.7	3.56	1	--	--
.045	12.77	15.7	3.96	1	--	--
.050	14.47	19.27	4.39	1	.99988	$-4.84\text{x}10^{-4}$

Now for $x < x_m$.

$$\overline{E}^2 = K\left[e^\eta - 1 + e^\eta erf\sqrt{\eta} - 2\left(\frac{\eta}{\pi}\right)^{1/2}\right]$$

So

$$\rho(x) = -\frac{K\varepsilon_o}{2}\left[e^\eta + \frac{2}{\sqrt{\pi}} + e^\eta erf\sqrt{\eta} - \frac{1}{\sqrt{\pi\eta}}\right]$$

$x(cm)$	ξ	η	$\left\|f_{\frac{1}{2}}(\eta)\right\|$	$\rho(coul/m^3)$
0	-2.509	6.9078	2.001×10^3	.9687

Now we estimate ρ from the slope of $\overline{E}$.

$$\rho = \varepsilon_o\frac{d\overline{E}}{dx} = \left(8.854\times10^{-12}\ F/m\right)\left(\frac{d\overline{E}}{dx}V/m^2\right)$$

$$= 8.854\times10^{-12}\left(\frac{d\overline{E}}{dx}\right)coul/m^3$$

The final table is

$x(cm)$	$\rho\left(coul/m^3\right)$
$3.125\text{x}10^{-4}$	-.1673
$9.375\text{x}10^{-4}$	-.01753
$1.875\text{x}10^{-3}$	-.005
$3.75\text{x}10^{-3}$	$-1.531\text{x}10^{-3}$
$5.5\text{x}10^{-3}$	$-7.17\text{x}10^{-4}$
$8\text{x}10^{-3}$	$-4.59\text{x}10^{-4}$
$2\text{x}10^{-2}$	$-1.61\text{x}10^{-5}$

3.5x10^{-2}	-6.9947x10^{-5}
4.5x10^{-2}	-7.119x10^{-5}

Now we determine the total space charge between the plates.

$$\varepsilon_o \frac{d\overline{E}}{dx} = \rho(x) \qquad coul/m^3$$

$$\varepsilon_o \int_{x_a}^{x_b} \frac{d\overline{E}}{dx} dx = \underbrace{\int_{x_a}^{x_b} \rho(x) dx}_{coul/m^2}$$

So

$$\varepsilon_o \left[\overline{E}(x_b) - \overline{E}(x_a)\right] = Q/A$$

Where x_a and x_b are arbitrary points, and A = cross-sectional area.

For $x < x_m$

$$\overline{E}^2 = K\left[e^{\eta} - 1 + e^{\eta} erf\sqrt{\eta} - 2\left(\eta/\pi\right)^{1/2}\right]$$

$$x = 0, \quad \overline{E}(0) = 1.4382 \times 10^2\ V/cm; \quad x = x_m \quad \overline{E}(x_m) = 0$$

So the charge between $x = 0$ and $x = x_m$ is

$$\varepsilon_o \left[-1.4382 \times 10^3\ V/cm \times 10^2 \frac{cm}{m}\right] = Q_1/A$$

$$A = .2cm^2 = 2 \times 10^{-5} m^2$$

So $Q_1 = \left(2 \times 10^{-5} m^2\right)\left(8.854 \times 10^{-12}\ F/m\right)\left(-1.4382 \times 10^5\ V/m\right)$

$$= -2.547 \times 10^{-11} \quad coul \quad \Rightarrow \quad 1.59 \times 10^8 e^-$$

For $x > x_m$

$$\overline{E}^2 = K\left[e^{\eta} - 1 - e^{\eta} erf\sqrt{\eta} + 2\left(\frac{\eta}{\pi}\right)^{1/2}\right]$$

At $x = x_m$

$$\overline{E}^2 = K[1-1-0+0] = 0$$

Hence $\overline{E}(x_m) = 0$ as before.

At $x = x_a$ (the anode) $\overline{E}(x_a) = -64\,{}^{V}\!/_{cm}$

$$\varepsilon_o\left[-64\,{}^{V}\!/_{cm} \times 10^2 \frac{cm}{m} - 0\right] = {}^{Q_2}\!/_{A}$$

$$\therefore Q_2 = -.1133 \times 10^{-11}\,coul \quad \Rightarrow \quad 7.08 \times 10^6\,e^-$$

Then the total charge is

$$Q_{TOT} = Q_1 + Q_2 = -2.66 \times 10^{-11}\,coul \quad \Rightarrow \quad 1.66 \times 10^8\,e^-$$

We may now ascertain the velocity field:

$$J = 50\,{}^{Amp}\!/_{m^2} = \rho(x)v(x)$$

$$\therefore v(x) = \frac{50}{\rho(x)} \quad m/\sec$$

The table is:

$x(cm)$	$v(m/\sec)$
3.125×10^{-4}	2.99×10^{2}
9.375×10^{-4}	2.85×10^{3}
1.875×10^{-4}	9.956×10^{3}
3.75×10^{-3}	3.27×10^{4}
5.5×10^{-3}	6.97×10^{4}
8×10^{-3}	1.089×10^{5}
2×10^{-3}	3.1×10^{5}
3.5×10^{-2}	7.148×10^{5}
4.5×10^{-2}	7.02×10^{5}

Note the thermal emission velocity is about 4.0E+5 *m*/sec. So the values in the above table are very reasonable. Notice the velocity varies by about a factor of 1000.

The magnitude of the electric field $\left|\overline{E}(x)\right|$, across the diode is shown below. The cathode value is 1,438.2 volts/cm, while that at the anode is –64 volts/cm. The field passes through zero at 0.074 mm from the cathode.

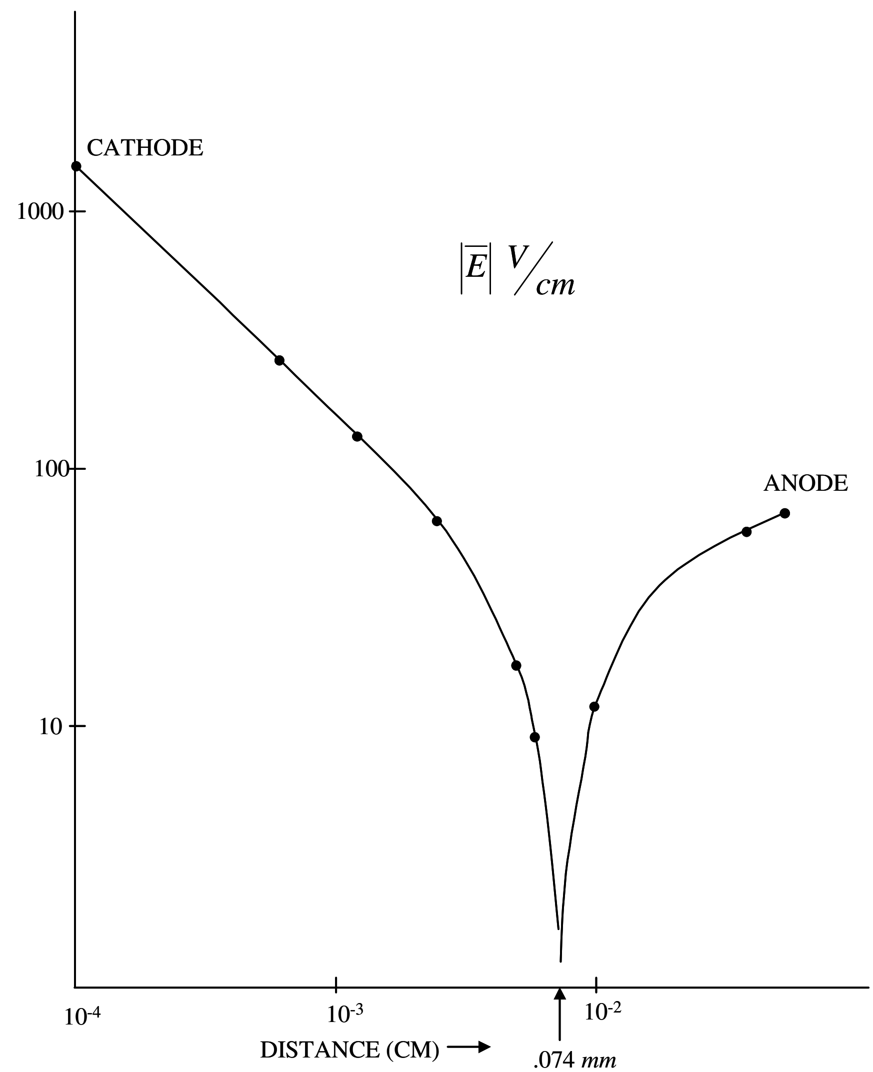

Figure 8 - A plot of $\left|\overline{E}\right|$ through the diode.

Below is the charge density $\rho(x)$ and the velocity field *v(x)* across the diode.

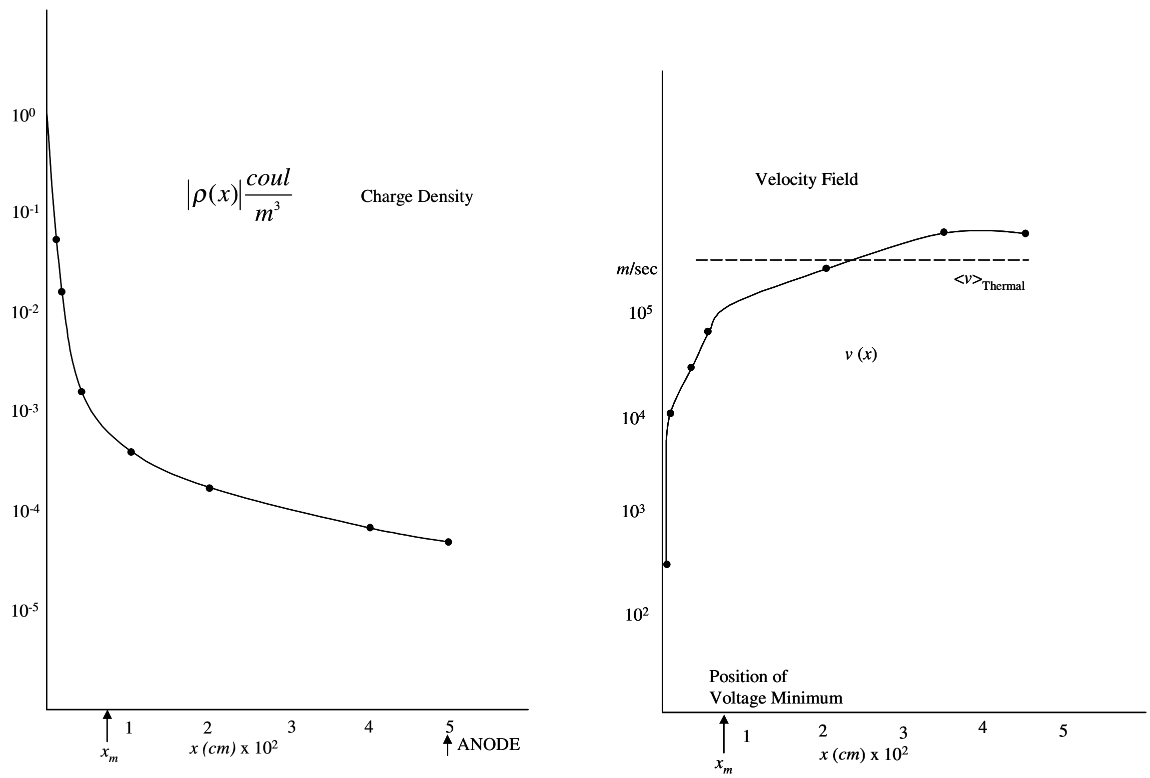

Figure 9 - The charge density $|\rho(x)|$ and velocity field $v(x)$ for the problem at hand.

This completes the calculation of a 1-dimensional diode in a regime where the 3/2 power law is not applicable. That regime is when J/J_o is between very roughly 0.01 to 0.5, rather than our case of 0.001. It is a useful example as the voltage minimum is away from the plane of the cathode and the charge density is not extremely large near the cathode. Here the density drops 3 orders of magnitude from the cathode to the voltage minimum (the virtual cathode), and the applied voltage is only 1.172 volts. We are drawing 1 *mA*, which is not too far from some actual TWTs (at least order of magnitude). Reference [11], by Kirstein, Kino, and Waters, has a good treatment of the problem, albeit in a notation that is unique. See refs. [12] through [15] for more information on this problem.

Chapter 3 Particle Enumeration

We now attempt to determine the way to model the 1-dimensional diode using a finite number of particles emitted over some specified time interval. First we divide the space between the plates into equal segments Δd, and over the segments we use $\Delta t = \Delta d / \Delta v$, where Δv is the change in the velocity field over a segment. By using 9 such equal length segments we ascertain the transit time is about 27 nsec (for the Low Voltage Diode studied earlier). Gewartowski [6], pg. 629 gives the time of flight for the 3/2 law regime as

$$t_f = \left[\frac{6\varepsilon_o d}{\eta J}\right]^{1/3}$$

which for our case is 1.45 nsec. The difference of more than an order of magnitude shows how far we are from the normal regime.

Recall, we would like to emit particles at the virtual cathode (plane of the voltage minimum). Help in this task is provided by [11] on pg.275, eq. (2.44). This expression gives the variation of potential with distance near the voltage minimum

$$V - V_m = \frac{\sqrt{\pi}1.82\times10^{-4}}{2\varepsilon_o\sqrt{T}} J\left(x - x_m\right)^2 \tag{3-1}$$

J in *Amp/m*^{2}

$$\frac{dV}{dx} = \frac{\sqrt{\pi}1.82\times10^{-4}}{2\varepsilon_o\sqrt{T}} J\left[2\left(x - x_m\right)\right] \tag{3-2}$$

The next derivative gives the charge density, which we see is not a function of position about the virtual cathode.

$$\frac{d^2V}{dx^2} = \frac{\sqrt{\pi}1.82\times10^{-4}}{\varepsilon_o\sqrt{T}} J \tag{3-3}$$

Observe eq. (3-2) shows that the electric field varies linearly through the virtual cathode plane. Then

$$\rho(x_m) = -\sqrt{\frac{\pi}{T}}1.82\times10^{-4}J$$
$$= -\sqrt{\frac{\pi}{1100}}1.82\times10^{-4}\left(50\,{}^{A}\!/_{m^2}\right) = -4.86\times10^{-4}\;{}^{c}\!/_{m^3}$$

Notice the analytical result in the section "Low Voltage Diode" gave the charge density of -7 x 10^{-4} coul/m^3, so our results show remarkable consistency. Since

$$\begin{aligned}\rho(x_m) &= -\sqrt{\frac{\pi}{T}}1.82\times10^{-4}\left[\langle -\rho(x_m)\rangle\langle v(x_m)\rangle\right]\\ \therefore \langle v(x_m)\rangle &= \sqrt{\frac{T}{\pi}}5.49\times10^{3}\,m/\mathrm{sec}\end{aligned} \tag{3-4}$$

Which states the average velocity at x_m is independent of all other parameters except the cathode temperature. For typical ranges of T we find

T(K)	v(x_m) m/sec
1000	9.79x10^4
1050	1.0x10^5
1100	1.03x10^5
1150	1.05x10^5
1200	1.07x10^5

So <v (x_m) > is nearly 1.0 x 10^5 m/sec for all temperatures and applied voltages, and currents. For reference, the velocity values in bins 1 and 14 are 1.29 x 10^5 and 6.71 x 10^5 m/sec.

Observe that the energy needed to overcome the retarding field of the space charge layer is

$$KE = q(0.655 volts) = 1.048 \times 10^{-19}\ Joules$$

The velocity required is

$$\frac{1}{2}mv^2 = 1.048 \times 10^{-19}$$

$$\therefore v = 4.8 \times 10^5\ m/\sec$$

So only those particles in bins 8-14 can escape the region and proceed on to the collector. This amounts to only 0.07 of the total number emitted per unit time.

Now consider a diode carrying the current i. Then the current in a given patch is

$$i_p = \frac{i}{N_p} = \frac{\Delta Q_p}{\Delta t} \tag{3-5}$$

Where N_p = the number of patches chosen for the entire cathode. Then

$$\Delta Q_p = i_p \Delta t = \sum n_p \Delta Q \tag{3-6}$$

Where n_p = the number of particles per patch, and $\sum \Delta Q$ is their net charge. Here ΔQ_p is the total charge emitted in a patch in the specified time interval Δt.

Now

$$\Delta Q_p = \Delta Q_1 + \Delta Q_2 + \cdots + \Delta Q_{14} \tag{3-7}$$

where

$$\Delta Q_1 = n_1 q_1, \ldots, \Delta Q_{14} = n_{14} q_{14} \tag{3-7a}$$

Since there are 14 energy bins. Here q_j is the charge value in a given bin. Consider bin_j; its total kinetic energy *(KE)* is

$$KE_j = n_j \left(\frac{1}{2} M_j \right) v_j^2 \tag{3-8}$$

Where n_j, M_j, v_j are the number of particles, their mass, and their common speed respectively. If the particles in all of the bins had the same mass m (this is the actual case in a real device), then

$$KE_{14} = n_{14}\left(\frac{1}{2}m\right)v_{14}^2 \tag{3-9}$$

$$KE_j = n_j\left(\frac{1}{2}m\right)v_j^2 \tag{3-10}$$

re-arrange

$$\frac{KE_j}{KE_{14}} = \frac{n_j}{n_{14}}\left(\frac{v_j}{v_{14}}\right)^2 \equiv \mu_j \tag{3-11}$$

Now let us use n_{14} particles in bin_j, but keep its KE the same.

$$KE_j = n_{14}\left(\frac{1}{2}M_j\right)v_j^2 \tag{3-12}$$

We retain the original KE by assigning the particles a new mass M_j
From eqns. (3-11) and (3-12)

$$\mu_j KE_{14} = K_j = n_{14}\left(\frac{1}{2}M_j\right)v_j^2 \tag{3-13a}$$

Or

$$\frac{n_j}{n_{14}}\frac{v_j^2}{v_{14}^2}\left(\frac{1}{2}m_{14}v_{14}^2 n_{14}\right) = n_{14}\left(\frac{1}{2}M_j\right)v_j^2 \tag{3-13b}$$

Or

$$M_j = \frac{n_j}{n_{14}}m_{14} \tag{3-14}$$

Now the ratios n_j/n_{14} are known from the M-B distribution, and for our chosen energy bins, they are:

$$
\begin{array}{ll}
n_1 / n_{14} = 114.66 & n_8 / n_{14} = 14.94 \\
n_2 / n_{14} = 132.13 & n_9 / n_{14} = 9.65 \\
n_3 / n_{14} = 104.38 & n_{10} / n_{14} = 6.19 \\
n_4 / n_{14} = 75 & n_{11} / n_{14} = 3.95 \\
n_5 / n_{14} = 51.76 & n_{12} / n_{14} = 2.51 \\
n_6 / n_{14} = 34.74 & n_{13} / n_{14} = 1.59 \\
n_7 / n_{14} = 22.93 &
\end{array}
\tag{3-15}
$$

Since MAFIA keeps the charge to mass ratio constant (equal to that of an electron),

$$\frac{q_j}{M_j} = \frac{q_{14}}{m_{14}}$$

Hence

$$q_j = \frac{M_j}{m_{14}} q_{14} \tag{3-16}$$

Returning to eq. (3-7) we observe

$$\Delta Q_1 = \frac{n_1}{m_{14}} q_{14} n_{14} \tag{3-16a}$$

And so forth; Thus

$$\Delta Q_p = \left(\frac{M_1}{m_{14}} + \frac{M_2}{m_{14}} + \cdots \frac{M_{13}}{m_{14}} + 1 \right) q_{14} n_{14} \tag{3-17}$$

Where we have set the number of particles in each bin to be n_{14}, as stated after eq. (3-11). Now from eq. (3-14) $M_j/m_{14} = n_j/n_{14}$, so the term in parentheses in eq. (3-17) is known

$$\left(\frac{n_1}{n_{14}} + \cdots + \frac{n_{13}}{n_{14}} + 1 \right) = 575.43 \tag{3-18}$$

Where we have used eq. (3-15). Thus eq. (3-17) becomes

$$\Delta Q_p = 575.43 q_{14} n_{14} = i_p \Delta t \tag{3-19}$$

Now eq. (3-19) is our basic working equation. The current from the patch i_p is known, the time interval Δt may be chosen compatible with MAFIA's constraints on run time, etc. The number of carriers may be chosen, then the charge on the particles (macroparticles in MAFIA) is determined. Eq. (3-19) determines the charge on the fast particles, which are in bin 14. The mass and charge for the remaining 13 bins are found using eqns. (3-14), (3-15), and (3-16a).

Before q_{14} is determined we must choose n_{14}. The total number of charges emitted during the interval Δt is

$$N_{TOTC} = n_{14}(14)N_p \tag{3-20}$$

Where N_{TOTC} is the total number emitted for all patches. For a given energy bin we know the particles are distributed as $cos\theta$, so we choose the following recipe. Let 1 particle emerge at 85° and let the number increase as $\theta \to 0$. The table is:

$\theta°$	# particles	$cos\theta$
85	1	0.0872
80	2	0.1737
75	3	0.2588
70	4	0.342
65	5	0.4226
60	6	0.500
55	7	0.574
50	7	0.643
45	8	0.707
40	9	0.766
35	9	0.8192

30	10	0.866
25	10	0.906
20	11	0.9397
15	11	0.9659
10	11	0.9848
5	11	0.99619
0	11	1.000

Then n_{14} = 136 particles/energy bin. The number of particles processed in a complete MAFIA run is

$$N_{TOT-RUN} = (136)(14) N_p \; N_{cyc} \tag{3-21}$$

Where N_{cyc} is the total number of emission cycle intervals.

We can apply this to our low voltage diode problem as follows. The total charge between the cathode and the voltage minimum is

$$Q_1 = -2.5468 \times 10^{-11}\; C \Rightarrow 1.5917 \times 10^{8} e^{-}$$

While that between the virtual cathode and the anode is

$$Q_2 = -.1133 \times 10^{-11}\; C \Rightarrow 7.0833 \times 10^{6} e^{-}$$

$$Q_{TOT} = Q_1 + Q_2 = -2.66 \times 10^{-11}\; C \Rightarrow 1.6625 \times 10^{8} e^{-}$$

Since the time of flight is about 27 nsec, choose Δt = 2.7 nseconds. Choose 10 patches, then $i_p = 10^{-4}$ Amp, and from (3-19)

$$\Delta Q_p = 10^{-4}(2.7 \times 10^{-9}) = 2.7 \times 10^{-13}\, C$$

So the amount of charge developed/emission cycle is

$$10 \Delta Q_p = 2.7 \times 10^{-12}\, C$$

The total charge between the plates is then generated in

$$\frac{2.66\times10^{-11}}{2.7\times10^{-12}}=9.85\,cycles$$

Also from (3-19), the charge q_{14} is

$$q_{14}=\frac{2.7\times10^{-13}C}{(575.43)(136)}=3.45\times10^{-18}C\Rightarrow 21.56e^{-}$$

If we use 50 cycles, the total number of particles processed is

$$N_{TOT-RUN}=(136)(14)(10)(50)=9.52\times10^{5}\ particles/run$$

So 9.52 x 10^5 particles emitted during 135 nsec. This value corresponds with that used by C. Kory and K. Vaden using the program. C. Kory uses about 2.8 x 10^5 particles per run.

At this point we have all the information to emit particles from the virtual cathode. We now must determine the correct M-B energy distribution at this plane. We know the saturation current is 10^3 times larger than that collected. Only 10^{-3} of that emitted by the actual cathode is actually collected; the remainder is pushed back into the cathode, only to be re-emitted again. In other words, as we move from the cathode toward the voltage minimum, the carrier density decreases rapidly, but the imbalance of carriers moving in opposite directions still maintains the net 1 mA current toward the anode. At the virtual cathode and beyond, particles are only moving toward the anode and they constitute the 1 *mA* current at every plane.

The velocity distribution for each bin was given in the Emission Modeling section, eq. (1-26) and subsequent table. Eq. (3-4) above shows the average velocity at the minimum to be

$$\langle v(x_m)\rangle=\sqrt{\frac{T}{\pi}}5.49\times10^{3}\,m/\sec \qquad (3\text{-}22)$$

And for our temperature, T = 1100° K, we have

$$\langle v(x_m) \rangle = 1.03 \times 10^5 \, m/\sec$$

From [8], the most probable speed occurs when $\bar{\eta}_j = 1$, (see eq. 8.32, pg 224). Then from eq. (1-26) of the Emission Modeling section we find the most probable speed = v_c = 2.582 x 10^5 m/sec. Then the average speed $\bar{v}$ is 1.128 v_c = 2.9125 x 10^5 m/sec. The rms value is 1.224 v_c = 3.16 x 10^5 m/sec. Also from eq. (1-26)

$$v_c = 7.785 \times 10^3 \sqrt{T} \qquad (\text{since } \bar{\eta}_j = 1) \tag{3-23}$$

So the average velocity is

$$\bar{v} = (1.128)(7.785 \times 10^3)\sqrt{T} \tag{3-24}$$

The above is for the particles leaving the cathode, what we need, however, is the velocity distribution at the plane of the voltage minimum (the virtual cathode). It is calculated as follows. We know the energy in a particular bin is $2\bar{\eta}_j kT_1$, and for bin 14 for our temperature it is 1.2808 *eV*. The factor of two occurs since we are considering those electrons that have escaped. The potential hill each particle must overcome is 0.655 *eV* (the voltage at the minimum is 0.655 volts). Thus, the fastest particles beyond the minimum have energy 1.2808 - .655 = .6258 *eV*. Now from experiments, we know the distribution is still Maxwell-Boltzmann, so this energy must correspond to those in bin 14 at the new, but lower, effective temperature. Bin 14 is characterized by $\bar{\eta}_j$ = 6.75. Thus our new temperature is defined by

$$\begin{aligned} .6258 &= (6.75)k\,Teff = \bar{\eta}_j kTeff \\ \therefore Teff &= 1075^\circ K \end{aligned} \tag{3-25}$$

Notice here the factor of 2 does not appear, as now we treat all emitted electrons as constituting a normal M-B gas. Thus our new temperature is 1075° K. We can always determine the effective temperature since the value of the voltage minimum is known once the total current is chosen, and the saturation current is known for the specific cathode. The minimum voltage is given by

$$V_m = -\frac{T}{11{,}605}\ln\left(\frac{J_o}{J}\right) \tag{3-26}$$

Where T is that for the cathode. Once the new temperature T_{eff} is known, the velocities for the bins become

$$v_j = 3.893\times10^3\sqrt{\bar{\eta}_j T_{eff}} \quad m/\sec \tag{3-27}$$

The new table for the velocities (at the voltage minimum) as compared to those in the Emission Modeling section (which were at the cathode), is:

Bin	$\bar{\eta}_j$	*v (m/sec)*
1	.25	$3.191\text{x}10^4$
2	.75	$9.573\text{x}10^4$
3	1.25	$1.596\text{x}10^5$
4	1.75	$2.234\text{x}10^5$
5	2.25	$2.872\text{x}10^5$
6	2.75	$3.5101\text{x}10^5$
7	3.25	$4.148\text{x}10^5$
8	3.75	$4.787\text{x}10^5$
9	4.25	$5.425\text{x}10^5$
10	4.75	$6.063\text{x}10^5$
11	5.25	$6.7\text{x}10^5$
12	5.75	$7.34\text{x}10^5$
13	6.25	$7.98\text{x}10^5$
14	6.75	$8.616\text{x}10^5$

To simplify the modeling, we may use a built-in feature of MAFIA; that being a parabolic distribution with chosen parameter. We will approximate the cosine distribution with the first two terms of its Taylor expansion, which is a parabolic representation.

$$\begin{aligned} &\cos\theta = 1 + a_2\theta^2 \qquad a_2 \stackrel{\Delta}{=} -.4967 \\ &\therefore p(\theta) = 1 - .4967\theta^2 \qquad 0 \le \theta \le \pi/2 \end{aligned} \tag{3-28}$$

Beam Velocity Profile

After a careful study of the experimental results on the beams created by a typical electron gun, we may make the following tentative statements. Firstly, the reported results are somewhat ambiguous, and secondly, they do not give the information in the form useful for our modeling effort. We desire the particle density and velocity at least over a specified plane normal to the axis of the beam. The reports give approximations that yield either velocity or charge density vs. radial value, but not both. Some studies have stated the current density falls off in a Gaussian manner from the beam center to the edge. Stating that the current density is Gaussian only tells us that the product of particle density and velocity are such. For our efforts, we need both quantities, not just their product. Some papers however, state that the beam is far from laminar (or Gaussian) and that rings of charge start at the edge and move through the beam to the center. It is not clear if the reported results are due to the particular electron gun used, or are typical of all guns.

With the state of affairs as such, a reasonable approximation for the beam velocity profile chosen by C. Kory is as follows. At the emission plane initialize particles such that their angle with the normal θ is given by

$$\tan\theta = \frac{u_t}{u_z} \tag{1}$$

Where u_t and u_z are the transverse and longitudinal velocities, respectively. The longitudinal velocity is constant, due solely to the anode potential. The transverse component is assumed to be Gaussian. Writing the above in more familiar notation

$$y = \tan^{-1}\left(\frac{x}{a}\right) \tag{2}$$

In essence we are seeking the probability density function for θ (or y), given the density for x (the transverse component u_t).

When $y > 0$

$$x = a \tan y$$

Let $g(x) = y$, then

$$g^1(x) = \frac{a}{a^2 + x^2}$$

Then the density function for y ($y = \theta$) is $f_Y(y)$,

$$f_Y(y) = \frac{f_x(a \tan y)}{a/(a^2 + x^2)} = \frac{a^2 + x^2}{a} f_x(a \tan y)$$

where $f_X(x)$ is the density of the random variable x ($x = u_t$)

If x is normally distributed

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

Then

$$f_Y(y) = \frac{a^2 + x^2}{a} \frac{1}{\sigma\sqrt{2\pi}} e^{-a^2 \tan^2 y/2\sigma^2} \tag{3}$$

When $y < 0 \quad x = a \tan y$

So eq. (3) is the same as before. In eq. (3)

$$x^2 = a^2 \tan^2 y$$

Hence

$$f_Y(y) = a \sec^2 y \frac{1}{\sigma\sqrt{2\pi}} e^{-a^2 \tan^2 y/2\sigma^2} \tag{4}$$

In most guns

$$\frac{u_t}{u_z} \sim 10^{-2} \qquad \therefore \frac{x}{a} = 10^{-2} = \tan y$$

$$\therefore y = \theta = .573^\circ$$

The standard deviation is

$$\sigma = \sqrt{\frac{kTc}{m}} = 1.29 \times 10^5 \sqrt{c}$$

$$c = \left(\frac{r_c}{r_{95}}\right)^2 \doteq \left(\frac{.24}{.045}\right)^2 = 28.44$$

$$\therefore \sigma = 6.88 \times 10^5 \, m/\sec$$

For typical values $\sigma \sim 2 \times 10^5 \, m/\sec, a \sim 10^7 \, m/\sec$

$$f_Y(\theta = .573^\circ) = 17.65$$

Therefore eq. (4) is the probability density function for the emission angle for a given small patch, for a typical electron gun under the assumption of eq. (1). A sketch of $f_Y(\theta)$ is below.

$$f_Y(\theta) = \frac{a \sec^2\theta}{\sigma\sqrt{2\pi}} e^{-a^2 \tan^2\theta / 2\sigma^2}$$

$$a = u_z = 10^7 \, m/\sec$$

$$\sigma = 1.3 \times 10^5 \left(\frac{r_c}{r_{95}}\right)$$

$$\approx 2 \times 10^5 \, m/\sec$$

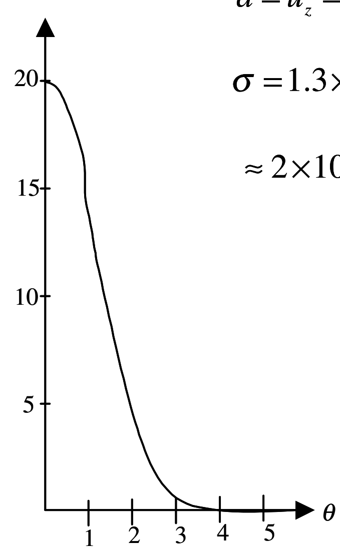

Figure 10 - The distribution $f_Y(\theta)$ for the emission angle θ from the normal.

Conclusions

It appears that MAFIA can be used to model electron guns with a Maxwell-Boltzmann energy distribution of the emitted electrons. To keep the run-time to a minimum, a scheme was devised that uses the same number of particles for any cathode current. The charge and mass of the particles are varied to conform to the given current. Similar to the older program EGUN, the calculation starts at a plane close to the actual cathode. In EGUN it is called the “starting surface” and is chosen somewhat arbitrarily (a few mesh cells from the cathode). EGUN sets up current tubes on the surface and varies them until the solution (forced to obey the Child-Langmuir 3/2 power law) is determined. We, however, emit charged particles (macroparticles) of varying charge and mass, which have the M-B energy distribution, and the proper distribution in emission angle from the cathode. Our emission plane is close to the virtual cathode whose position is known once the current is chosen. The actual virtual cathode varies with distance from the true cathode; it depends on the local cathode loading on a given patch. The loading varies by as much as 20% in some cases. The effective temperature of the macroparticles is calculated, given the known temperature of the cathode. The modeling of the beam should be closer to reality than any published scheme to date. The solution is not constrained to obey the 3/2 power law, so all regimes of emission are able to be examined. Specifically, if some region of the cathode is in saturation, this scheme can handle it easily. Also the non-laminar (noise) properties of the beam should be modeled completely. The use of particles will determine the true beam diameter as it exits the gun, as well as the spread of trajectories at the beam waist (near the point where the PPM stack applies the confining magnetic field).

The following bulletized list summarizes the accomplishments that are distributed throughout the memorandum.

- Explained how the program EGUN operates, and some of its shortcomings. Fundamentally, the procedure bypasses the establishment of the space-charge layer, from which the anode current is developed. Instead, it models the beam as current tubes. The tubes cannot cross one another; so laminar flow is implied, even though experiments suggest otherwise. The noise properties of the beam are therefore lost completely. The calculation assumes the 3/2 law, which eliminates the possibility of portions of the cathode being in saturation. A consequence of the assumptions is its difficulty in correctly determining the beam diameter along the gun, as well as finding the correct value of the space-charge along the axis. An empirical factor of 5.5 is used to correct the value found by the calculation. It does, however, find the perveance to a few per cent.
- Started with Lambert's law as given in Gewartowski [6]

  $$J(\theta) = \frac{J_o}{\pi}\cos\theta = qn(\theta)v(\theta) \doteq q\sum n_j(\theta)\,v_j(\theta)$$

  and showed that $n_j(\theta)$ is distributed as $\cos\theta$ for a fixed value of velocity v_j. This result followed from the statement that the kinetic energy normal to the surface is twice that tangential to the surface. This was proven in Appendix I, and developed in Chapter 1, Emission Modeling.
- With particle use, we will be able to model all parts of the I-V plane, as well as noise, and non-laminar flow.
- Chapter 1, Emission Modeling, indicates the basic way the problem should be handled. It ends by showing the numbers needed are too large for MAFIA. It demonstrated the difficulties encountered in trying to establish

the space-charge layer. It is actually this layer which serves to emit particles that form the beam.

- It was recognized that the emission maybe viewed as follows. There is a M-B gas in the cathode, and there is another M-B gas, at a different temperature, which forms the space-charge layer. Then we can define a third temperature T_{eff}, which characterizes those electrons that have traversed the space charge layer and are going toward the anode. Those that are collected are also M-B which was determined from the experimental results of references [1], and [2]. Appendix I also showed the escaping electrons to have a temperature such that the average energy was $2kT$ vs the $3kT/2$ of those that remain in the metal. Here T is the cathode temperature. However, in that calculation, the repelling force of the space-charge layer at the surface was not considered.
- Chapter 2 detailed the solution for the 1-dimensional diode that used the thermal velocity of the emitted charge as a boundary condition. The equations are non-linear, and tables have been established to facilitate the voltage, charge, electric field, and velocity field between the plates. We followed the lead of Beck [9] in this regard.
- We stated the boundary conditions for the 3/2 law require the charge density to be infinite at the virtual cathode plane, which is not possible to be modeled using particles.
- In principle, one should use particles in the following manner. With the cathode temperature given, the thermal limited current density is established for the particular cathode material. Emit particles with the M-B distribution, and which satisfy the current density constraint. The space-charge layer should form in the simulation, and steady state should occur. However, in

the middle range of the I-V characteristic, about ½ of those emitted will return to the cathode, only to be re-emitted. This would be a waste of computational time, just to establish this "bias condition" to allow the actual current to the anode to be established. Thus one must somehow sidestep the formation of the space-charge layer.

- Chapter 3 outlined a method to sidestep the formation of the space-charge layer. The idea is to only emit those that reach the anode. The determination of their effective temperature T_{eff} was presented.
- Chapter 1 presented some bunches (17 angle sets) to be emitted if a distribution function in the program MAFIA was found to not be appropriate. A function $f(\theta)$ was developed to determine the correct makeup of a given bunch.
- We plan to model the cosine distribution for each energy bin group with a parabolic distribution function which is available in MAFIA.
- Chapter 2 gave a solution for 1 *mA* of current and 1.172 volts. There, representative numbers were determined. In an actual gun, say the one in the Cassini Mission TWT, the current is 15 *mA* and the voltage is 5*kV*. Thus we must verify the model using the 1.172 case.
- For the example we found the charge density varies by 4 orders of magnitude through the diode. The electric field changed from + to -, and varied by a factor of 22 from cathode to anode. The velocity field changed by a factor to 1000. Finally, the total number of charges between the plates in steady state is 166 million. Of these, 159 million are between the cathode and the plane of the voltage minimum.
- Chapter 3 explained the method to keep the total number of particles in a given run constant. The variation in current is handled by adjusting the

charge and mass of the particles in the fixed 14 energy bins. This will dovetail into the program's requirement of 4-5 macroparticles per mesh cell.

- We used [11] to realize the average velocity near the voltage minimum is nearly constant for all current levels; it varies only slightly with cathode temperature. Looking back at the numerical solution obtained in Chapter 2 we found complete compliance with this result!
- With all of the compression done on the number of particles used, the runs may take 12-18 hours. We hope we have over-estimated the run times.
- We have developed the probability distribution of the angle θ from the beam axis, $f_Y(\theta)$, for a reasonable approximation for defining an electron beam for other uses in parts of TWT simulations. If the transverse velocities are gaussian, while the normal component is constant, we find θ is distributed as

$$f_Y(\theta) \equiv f_\Theta(\theta) = u_z \sec^2\theta \frac{1}{\sigma\sqrt{2\pi}} e^{-u_t^2/2\sigma^2}$$

Appendix I

Section 5-15 of [8], page 141, concerns itself with the energies of escaping electrons from a hot cathode. This is an important result, as most formulas, results, concepts, etc. from electron gas theory in solids is for the case where the carriers remain in the metal. While we know the statistics of the ensemble for the electrons in the metal, what are they for the portion that escapes? It is more convenient to attack this problem as a function of velocity, thus

$$qE = \frac{1}{2}mv^2$$

Notice the notation that electric potential is given the symbol E (a peculiarity of Millman and Seely), while in Gewartowski it is given the label W. We use this notation since the development to follow is from Millman and Seely. Our goal is to show

$$KE_{normal} = 2 \times KE_{\tan gential} \qquad \text{(I-1)}$$

The calculation proceeds by determining the average energy of those electrons in a metal that are capable of escaping. This is eq. (5-42)

$$\overline{E} = \frac{\int_{v_x=v_{xB}}^{\infty}\int_{v_y=-\infty}^{\infty}\int_{v_z=-\infty}^{\infty}\frac{1}{2}\frac{m}{q}\left(v_x^2+v_y^2+v_z^2\right)dN_t^1}{\int_{v_x=v_xB}^{\infty}\int_{v_y=-\infty}^{\infty}\int_{v_z=-\infty}^{\infty}dN_t^1} \qquad \text{(I-2)}$$

After some calculations the following expression is found

$$\int_{v_xB}^{\infty}\frac{1}{2}\frac{m}{q}v_x^3e^{-\lambda v_x^2}dv_x \Big/ \int_{v_xB}^{\infty}v_xe^{-\lambda v_x^2}\,dv_x$$

Evaluating the integral in the numerator yields; let $\frac{m}{2q} = k_1$

$$\int_{v_xB}^{\infty}k_1v_x^3e^{-\lambda v_x^2}dx \qquad let \quad u = v_x^2, \quad du = 2v_xdv_x$$

then

$$-\frac{k_1}{2}\frac{e^{-\lambda u}}{\lambda}\left(u+\frac{1}{\lambda}\right)_{u_B}^{\infty}$$

$$=-\frac{k_1}{2\lambda}\left[-e^{-\lambda u_B}\left(u_B+\frac{1}{\lambda}\right)\right]$$

$$=\frac{k_1}{2\lambda}e^{-\lambda v_{xB}^2}\left(v_{x_B}^2+\frac{1}{\lambda}\right) \tag{I-3}$$

$$\lambda=\frac{m}{2qE_T}$$

Their expression in the denominator is

$$\int_{v_{x_B}}^{\infty} v_x e^{-\lambda v_x^2} dv_x$$

and its evaluation proceeds as (a standard form)

$$=\frac{1}{2\lambda}e^{-E_B/E_T} \qquad E_B=Min.\ of\ E \tag{I-4}$$

The ratio is

$$k_1 e^{-\lambda v_{xb}^2+E_B/E_T}\left(v_{xB}^2+\frac{1}{\lambda}\right) \tag{I-5}$$

Now

$$\lambda v_{xB}^2=\frac{m}{2qE_T}v_{xB}^2$$

if we assume

$$E_B=\frac{1}{2q}mv_{xB}^2 \tag{I-6}$$

then

$$e^{-E_B/E_T+E_B/E_T}=1$$

Then the ratio is

$$kv_{xB}^2 + k/\lambda = \frac{1}{2}\frac{m}{q}v_{xB}^2 + \frac{1}{2}\frac{m}{q}\frac{2qE_T}{m} = E_B + E_T \tag{I-7}$$

as stated. The second term of eq. (5-42) is

$$\frac{\iiint\left(\frac{1}{2}\frac{m}{q}v_y^2\right)\left(\frac{2m^3}{h^3}e^{E_M/E_T}\right)e^{-\lambda v_x^2-\lambda v_y^2-\lambda v_z^2}v_x dv_x dv_y dv_z}{\iiint\frac{2m^3}{h^3}e^{E_M/E_T}e^{-\lambda v_x^2-\lambda v_y^2-\lambda v_z^2}v_x dv_x dv_y dv_z}$$

$$= \frac{\left[\int_{v_xB}^{\infty}e^{-\lambda v_x^2}v_x dv_x\right]\int_{-\infty}^{\infty}k_1 v_y^2 e^{-\lambda v_y^2}dv_y}{\left[\int_{v_xB}^{\infty}e^{-\lambda v_x^2}v_x dv_x\right]\int_{-\infty}^{\infty}e^{-\lambda v_y^2}dv_y} \tag{I-8}$$

$$= k_1\frac{\int_{-\infty}^{\infty}v_y^2 e^{-\lambda v_y^2}dv_y}{\int_{-\infty}^{\infty}e^{-\lambda v_y^2}dv_y} = k_1\frac{(2)\left(\frac{1}{4}\sqrt{\pi/\lambda^3}\right)}{(2)\left(\frac{1}{2}\sqrt{\pi/\lambda}\right)} = k_1\frac{1}{2\lambda}$$

$$= E_T/2$$

as stated. The final statements are: In leaving the metal, the average X-directed energy of each electron is reduced by an amount E_B, so that the average X-directed energy of each electron *after it has left the metal is* $(E_B + E_T) - E_B = E_T$ electron volts. Hence the total average energy of each electron that *escapes from the metal* is $E_T + 1/2E_T + 1/2E_T = 2E_T$ electron volts. In other words the normal is E_T whereas the Y-directed (which is tangential here) is $(1/2)E_T$. This is our first equation in this Appendix.

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List of Symbols

I	d.c. gun (cathode) current
K_p	perveance of electron gun
J	current density (amps/m^2)
V	voltage (or potential) along gun or 1-dimensional diode
$\overline{E}$	electric field in 1-dimensional diode
T_1	cathode temperature
T_{eff}	effective temperature of the M-B ensemble that constitutes the anode current
M-B	Maxwell-Boltzmann energy distribution
J_o	saturation current density at a given temperature for a specific cathode material
θ	angle from the normal to an emitting patch
φ	azimuth angle
v	particle velocity
< >	average of quantity over the ensemble
<v>	average velocity of the particles over the ensemble
W_T	thermal equivalent of energy
m	electron mass
k	Boltzmann's constant
q	magnitude of electron charge
n_p (θ)	number of particles emitted vs. θ (number/m^3)
v(θ)	velocity of particles emitted vs. θ (m/sec)
U.S.A	unit solid angle (differential solid angle = 2π sinθ dθ)
dP (u, θ)	joint probability of emission velocity u and direction θ
dN_E	number of particles with energy in the range E + dE

N_{TOT}	total number of particles in ensemble (or a density:/m^3)
E_T	W_T energy (in Millman's [8] notation). Note [8] uses E for both energy and voltage, a very confusing choice, even though they attempt to justify it in the beginning chapter
ρ_η	$dN_\eta / d\eta$ Particle density with respect to the normalized energy η
η	E/E_T normalized energy parameter
E	a constant = kT (Joules) or kT/q electron volts; for a temperature T
I_{PATCH}	current from a patch of cathode
A_{PATCH}	area of patch
n_j	particle density in energy bin_j
$\propto_j$	constant, j = 1,…,B
B	number of energy bins
KE_{normal}	total kinetic energy normal to the surface
$KE_{tangential}$	total kinetic energy parallel to the surface
n_{ji}	particle density in bin_j with velocity at angle θ_i
κ_j	a constant
$f(\theta)$	$\frac{1}{2}\cos^3\theta - \cos\theta\sin^2\theta$
erf	error function
$\bar{\eta}_j$	E_j/E_T normalized energy parameter for bin_j
η	normalized parameter in the 1-dimensional diode; not to be confused with η for normalized energy
η_c	η parameter at cathode
ξ	normalized distance parameter defined by Langmuir [10]
$\xi_{c,a}$	normalized parameter at cathode or anode
d	separation of planes in 1-dimensional diode

V_m	voltage at minimum (virtual cathode plane)
V_a	voltage at anode
K	constant in analytical expression for $\overline{E}$
$\rho(x)$	charge density (Coulombs/m^3)
ε_o	permittivity of free space
A	cross-sectional area
x_a	position of anode
v(x)	velocity field in the 1-dimensional diode
TWT	traveling wave tube
t_f	time of flight of a particle in the 3/2-law regime
i_p	patch current
Np	number of patches
ΔQ_p	total charge emitted from a patch during Δt
ΔQ_j	net charge in bin_j
n_p	number of particles/patch emitted during Δt
q_j	charge of particles in bin_j
M_j	mass of particles in bin_j
m	mass (generally an electron)
n_j	number of particles (per m^3) in bin_j
u_t	transverse velocity in electron beam
u_z	longitudinal velocity in electron beam
EGUN	popular electron gun program (used for last 25 years)
PPM	periodic permanent magnet (stack)

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13. ABSTRACT ***(Maximum 200 words)***

This memorandum summarizes a study undertaken to apply the program MAFIA to the modeling of an electron gun in a traveling wave tube (TWT). The basic problem is to emit particles from the cathode in the proper manner. The electrons are emitted with the classical Maxwell-Boltzmann (M-B) energy distribution; and for a small patch of emitting surface; the distribution with angle obeys Lambert's law. This states that the current density drops off as the cosine of the angle from the normal. The motivation for the work is to extend the analysis beyond that which has been done using older codes. Some existing programs use the Child-Langmuir, or 3/2 power law, for the description of the gun. This means the current varies as the 3/2 power of the anode voltage. The proportionality constant is termed the perveance of the gun. This is limited, however, since the 3/2 variation is only an approximation. Also, if the cathode is near saturation, the 3/2 law definitely will not hold. In most of the older codes, the electron beam is decomposed into current tubes, which imply laminar flow in the beam; even though experiments show the flow to be turbulent. Also, the proper inclusion of noise in the beam is not possible. These older methods of calculation do, however, give reasonable values for parameters of the electron beam and the overall gun, and these values will be used as the starting point for a more precise particle-in-cell (PIC) calculation. To minimize the time needed for a given computer run, all beams will use the same number of particles in a simulation. This is accomplished by varying the mass and charge of the emitted particles (macroparticles) in a certain manner, to be consistent with the desired beam current.

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